MATH 141 - Quiz Solutions(12 noon), Fall 2009

1. Find the numerical value of $\sec(\sin^{-1}(\frac{\sqrt{3}}{2}))$.

Solution: Let $x = \sin^{-1}(\frac{\sqrt{3}}{2})$, then we know that $\sin(x) = \frac{\sqrt{3}}{2}$, and we want to find $\sec(x)$. Use the graph on the next page, and we find

$$\sec(x) = \frac{2}{1} = 2$$

2. Find the length of the curve described parametrically by $x = 1 - t^2$ and $y = 1 + t^3$, for $t \in [0, 2]$.

Solution: Use the formula

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

it follows that

$$\begin{aligned}
L &= \int_0^2 \sqrt{(2t)^2 + (3t^2)^2} dt \\
&= \int_0^2 \sqrt{9t^4 + 4t^2} dt \\
&= \int_0^2 t\sqrt{4 + 9t^2} dt \\
&\left(\text{substitution } u = 4 + 9t^2 \Rightarrow du = 18t dt\right) \\
&= \frac{1}{18} \int_4^{40} \sqrt{u} du \\
&= \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^{40} \\
&= \frac{1}{27} \left(40^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)
\end{aligned}$$

3. Find the formula for the inverse of the function $f(x) = -4x^3 - 1$

Solution: Let $y = -4x^3 - 1$, then

$$-4x^3 = 1 + y \Rightarrow x = \sqrt[3]{\frac{1}{4}(-1-y)} \Rightarrow f^{-1}(x) = \sqrt[3]{\frac{1}{4}(-1-x)}$$

4. Find the integral

$$\int e^{ex} dx$$

Solution:

$$\int e^{ex} dx = \frac{1}{e} e^{ex} + C$$

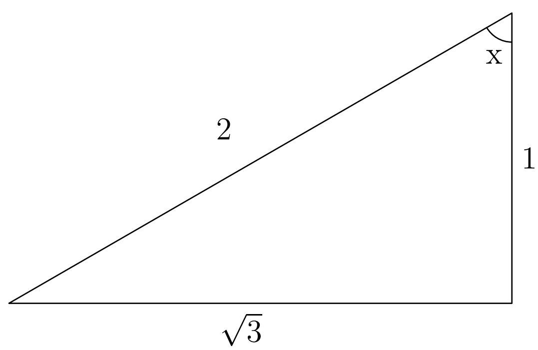

Figure 1: Graph for Problem 1

MATH 141 - Quiz Solutions (1pm), Fall 2009

1. Find the numerical value of $\sin(\sec^{-1}(\sqrt{3}))$.

Solution: Let $x = \sec^{-1}(\sqrt{3})$, then we know that $\sec(x) = \sqrt{3}$, and we want to find $\sin(x)$. Use the graph on the next page, and we find

$$\sin(x) = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

2. Find the length of the curve described parametrically by $x = 1 - t^3$ and $y = 1 + t^2$, for $t \in [0, 2]$.

Solution: Use the formula

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

it follows that

$$\begin{aligned}
L &= \int_0^2 \sqrt{(3t^2)^2 + (2t)^2} dt \\
&= \int_0^2 \sqrt{9t^4 + 4t^2} dt \\
&= \int_0^2 t\sqrt{4 + 9t^2} dt \\
&\left(\text{substitution } u = 4 + 9t^2 \Rightarrow du = 18t dt\right) \\
&= \frac{1}{18} \int_4^{40} \sqrt{u} du \\
&= \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^{40} \\
&= \frac{1}{27} \left(40^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)
\end{aligned}$$

3. Find the formula for the inverse of the function $f(x) = -4x^3 + 1$

Solution: Let $y = -4x^3 + 1$, then

$$4x^3 = 1 - y \Rightarrow x = \sqrt[3]{\frac{1}{4}(1-y)} \Rightarrow f^{-1}(x) = \sqrt[3]{\frac{1}{4}(1-x)}$$

4. Find the integral

$$\int e^{\sqrt{2}x} dx$$

Solution:

$$\int e^{\sqrt{2}x} dx = \frac{1}{\sqrt{2}} e^{\sqrt{2}x} + C$$

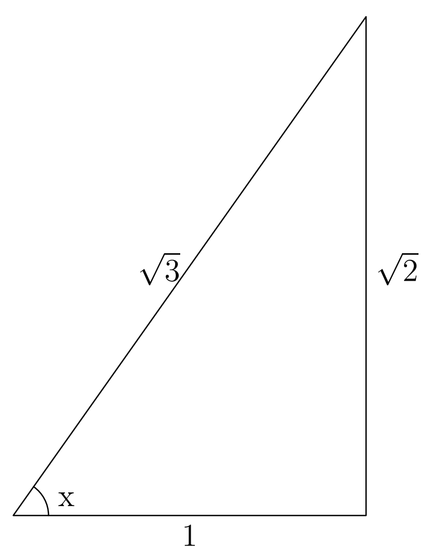

Figure 1: Graph for Problem 1

MATH 141 - Quiz Solutions(2pm), Fall 2009

1. Find the numerical value of $\csc(\tan^{-1}(\sqrt{3}))$.

Solution: Let $x = \tan^{-1}(\sqrt{3})$, then we know that $\tan(x) = \sqrt{3}$, and we want to find $\csc(x)$. Use the graph on the next page, and we find

$$\csc(x) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

2. Find the length of the curve described parametrically by $x = -1 + t^3$ and $y = 1 + t^2$, for $t \in [0, 4]$.

Solution: Use the formula

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

it follows that

$$\begin{aligned}
L &= \int_0^4 \sqrt{(3t^2)^2 + (2t)^2} dt \\
&= \int_0^4 \sqrt{9t^4 + 4t^2} dt \\
&= \int_0^4 t\sqrt{4 + 9t^2} dt \\
&\big(\text{substitution } u = 4 + 9t^2 \Rightarrow du = 18t dt\big) \\
&= \frac{1}{18} \int_4^{148} \sqrt{u} du \\
&= \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^{148} \\
&= \frac{1}{27} \left(148^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)
\end{aligned}$$

3. Find the formula for the inverse of the function $f(x) = -x^3 + 4$

Solution: Let $y = -x^3 + 4$, then

$$x^3 = 4 - y \Rightarrow x = \sqrt[3]{4 - y} \Rightarrow f^{-1}(x) = \sqrt[3]{4 - x}$$

4. Find the integral

$$\int \sqrt{e^x} dx$$

Solution:

$$\int \sqrt{e^x} dx = \int e^{\frac{x}{2}} dx = 2e^{\frac{x}{2}} + C$$

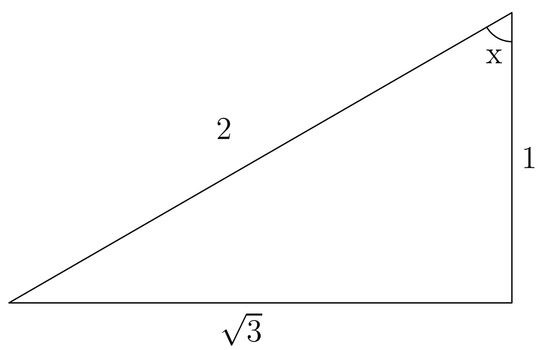

Figure 1: Graph for Problem 1

MATH 141 - Quiz Solutions (3pm), Fall 2009 Name ____________________

1. Find the numerical value of $\tan(\csc^{-1}(\sqrt{3}))$.

> **Solution:** Let $x = \csc^{-1}(\sqrt{3})$, then we know that $\csc(x) = \sqrt{3}$, and we want to find $\csc(x)$. Use the graph on the next page, and we find
>
> $$\tan(x) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

2. Find the length of the curve described parametrically by $x = -1 + t^2$ and $y = 1 - t^3$, for $t \in [0, 4]$.

> **Solution:** Use the formula
>
> $$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$
>
> it follows that
>
> $$\begin{aligned} L &= \int_0^4 \sqrt{(3t^2)^2 + (2t)^2} dt \\ &= \int_0^4 \sqrt{9t^4 + 4t^2} dt \\ &= \int_0^4 t\sqrt{4 + 9t^2} dt \\ &\left(\text{substitution } u = 4 + 9t^2 \Rightarrow du = 18tdt\right) \\ &= \frac{1}{18} \int_4^{148} \sqrt{u} du \\ &= \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^{148} \\ &= \frac{1}{27} \left(148^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) \end{aligned}$$

3. Find the formula for the inverse of the function $f(x) = -x^3 - 4$

Solution: Let $y = -x^3 - 4$, then

$$-x^3 = 4 + y \Rightarrow x = \sqrt[3]{-4-y} \Rightarrow f^{-1}(x) = \sqrt[3]{-4-x}$$

4. Find the integral

$$\int (e^x)^2 \, dx$$

Solution:

$$\int (e^x)^2 \, dx = \int e^{2x} dx = \frac{1}{2} e^{2x} + C$$

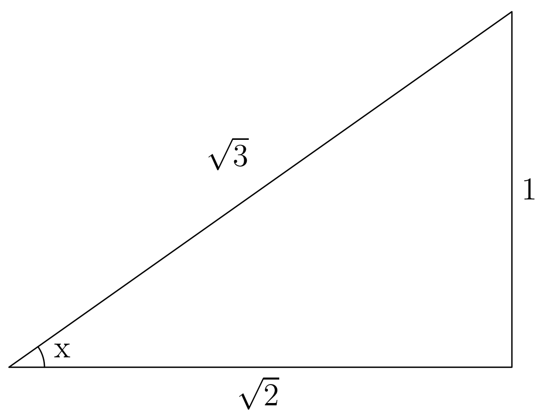

Figure 1: Graph for Problem 1