MATH 141, FALL 2008, Final practice

1) Find the length of the graph of $f(x) = e^x + e^{-x}$, $0 \le x \le 1$.

2) Find the area of the surface generated by revolving about the $x$-axis the curve with the following parametric representation: $x(t) = t^2$, $y(t) = t$, $1 \le t \le 4$.

3) Let $R$ be the region between the graphs of $f(x) = x^3$ and $g(x) = x^2$ on $[0, 1]$. Find the center of gravity of $R$ and the volume of the solid obatined by revolving $R$ about the $y$-axis.

4) A tank containing $144\pi$ cubic feet of water has the shape of a cone with vertex at the botom, with height of 20 feet and radius of 10 feet. Find the work needed to pump all the water to a point 10 feet above the tank.

5) Find a maximal interval on which $f(x) = x^3$ has an inverse.

6) Find the limit

$$\lim_{x \to 0} \frac{\arcsin(x)}{x}.$$

7) Solve the differential equation:

$$\frac{dy}{dx} + y \sin(x) = \sin(x).$$

8) Find the integral

$$\int \frac{\cos(x)}{1 + \sin^2(x)} \, dx.$$

9) Find the integral

$$\int x^2 \log_2(x^3) \, dx.$$

10) Determine whether the improper integral converges. If it does, determine its value:

$$\int_{-\infty}^{-2} \frac{1}{x^2 + 4} \, dx.$$

11) Find the interval of convergence of the series:

$$\sum_{n=2}^{\infty} (-1)^n \frac{(\log_2(n))^2}{n^2} x^n.$$

Explain which modes of convergence appear.

12) Determine whether the series converges

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}.$$

13) Find the Taylor series of $f(x) = \sin(x)$ at $a = \pi/4$.

14) Find all 3rd roots of $i$.

15) Find the length of the polar curve $r = 2\cos(\theta)$ and the area bounded by this curve.