

MATH 410, HW 1

1. Let r denote a rational real number and let x denote an irrational real number. Prove that $r - x$ and r/x are both irrational numbers.

2. Prove that there is no rational number r for which $r^2 = 12$.

3. Prove that between any two different rational numbers there exist infinitely many irrational numbers.

4. Find the limit

$$\lim_{n \rightarrow \infty} (\sqrt{n^2 + n} - n).$$

5. Assume that $\lim_{n \rightarrow \infty} a_n b_n = 0$. Does that imply that either $\lim_{n \rightarrow \infty} a_n = 0$ or $\lim_{n \rightarrow \infty} b_n = 0$? Justify your answer.