MATH 416, HW 1

1. Describe the sets of vectors $x \in \mathbb{R}^2$, for which $\|x\|_p = r$, for any $r > 0$, where $p = 1, 2, \infty$. Use this description to find a vector $z \in \mathbb{R}^2$ such that $\|z\|_2 = 1$ and $\|z\|_1$ is as large as possible. What is this maximal value of $\|z\|_1$?

2. For $x \in \mathbb{R}^N$ and $0 < p < 1$, define the following function: $F_p(x) = (|x_1|^p + \ldots + |x_N|^p)^{1/p}$. Is any of these functions a norm for $\mathbb{R}^2$? Justify your answer.

3. Write a computer program that takes as an input a natural number with up to 10 digits, and returns its binary representation.

4. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $e_1 = (0, 2, -2, 0)$, $e_2 = (0, 1, 0, -1)$, $e_3 = (0, -1, 1, -1)$.

5. Generate a random 6×8 matrix using, e.g., rand(6,8) command in MATLAB, or similar. Describe the linear span of the column and row vectors, respectively. Are the column (resp., row) vectors linearly independent?