MATH 416, HW 2

1. We say that a collection of vectors $\{f_1, \ldots, f_n\} \subset \mathbb{R}^d$, $n \geq d$ is a *strong finite frame* for $\mathbb{R}^d$ if there exist constants $0 < A < B < \infty$ such that for every vector $x \in \mathbb{R}^d$ the following holds:

$$A\|x\|^2 \leq \sum_{k=1}^{n} |\langle x, f_k \rangle|^2 \leq B\|x\|^2.$$

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Prove that the two definitions coincide, i.e., prove that every strong finite frame is a finite frame, and, vice versa, that every finite frame is a strong finite frame.

2. Implement in Matlab the 32×32 DFT as a matrix multiplication. Apply it to the following vectors: $v_1(k) = \sin(2\pi k/32), k = 0, \ldots, 31$, $v_2(k) = \sin(4\pi k/32), k = 0, \ldots, 31$, $v_3(k) = \cos(2\pi k/32), k = 0, \ldots, 31$. Plot the results in form of a function graph. Draw conclusions.

3. Implement in Matlab the 32×32 DFT by means of FFT algorithm. Apply it to the following vectors: $v_1(k) = \sin(2\pi k/32), k = 0, \ldots, 31$, $v_2(k) = \sin(4\pi k/32), k = 0, \ldots, 31$, $v_3(k) = \cos(2\pi k/32), k = 0, \ldots, 31$. Plot the results in form of a function graph. Draw conclusions.

4. Show that the vectors $\omega_n \in \mathbb{C}^N$ defined as $\omega_n(k) = \frac{1}{\sqrt{N}} e^{2\pi i n k/N}$ form an orthonormal basis for $\mathbb{C}^N$ with respect to inner product

$$\langle v, w \rangle = \sum_{k=1}^{N} v(k)\overline{w(k)}.$$