MATH 464, HW 7

1) One way to define an anlogue of a DFT that can be applied to a matrix (or an image) is through the following formula:

$$A[m,n] = \frac{1}{N^2} \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} a[j,k] e^{-2\pi i (j,k)\cdot(m,n)/N},$$

where $\{a[j,k]\}_{j,k=0}^{N-1}$ is the input $N \times N$ matrix, $(j,k)\cdot(m,n)$ is the inner product (i.e., $jm + kn$), and $\{A[j,k]\}_{j,k=0}^{N-1}$ is the output $N \times N$ matrix.

Compare theoretically this two-dimensional DFT transform to the ones described in class (sequentially applying 1-d DFTs to rows, and then columns; and vice versa, applying 1-d DFTs to columns, and then rows).

2) Implement in Matlab any two-dimensional DFT algorithm. Apply it to the following matrices: $a[m,n] = \sin(2\pi m/N)$, $m,n = 0,\ldots,N-1$, $N = 128$, $a[m,n] = \sin(4\pi m/N)$, $m,n = 0,\ldots,N-1$, $N = 128$, $a[m,n] = \sin(8\pi m/N)$, $m,n = 0,\ldots,N-1$, $N = 128$, $a[m,n] = \sin(2\pi m/N)\sin(2\pi n/N)$, $m,n = 0,\ldots,N-1$, $N = 128$. For comparison, apply this two-dimensional DFT to your favourite image.