8 AM

Question 1

Use the disk method. Then

$$\begin{aligned} V &= \int_0^{\pi/2} \pi(\sqrt{\cos(x)})^2 dx \\ &= \int_0^{\pi/2} \pi \cos(x) dx \\ &= \pi \sin(x)|_0^{\pi/2} \\ &= \pi \end{aligned}$$

Question 2

$$\begin{aligned} L &= \int_1^5 \sqrt{1 + f'(x)^2} dx \\ &= \int_1^5 \sqrt{5} dx \\ &= 4\sqrt{5} \end{aligned}$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_2^3 2\pi x(f(x) - g(x)) dx \\ &= \int_2^3 2\pi x(1 - (x - 2)) dx \\ &= \int_2^3 6\pi x - 2\pi x^2 dx \\ &= 3\pi x^2 - \frac{2}{3}\pi x^3|_2^3 \\ &= \frac{7\pi}{3} \end{aligned}$$

Question 4

$$\begin{aligned} L &= \int_0^1 \sqrt{1+f'(x)^2}dx \\ &= \int_0^1 \sqrt{1+(2x^{1/2})^2}dx \\ &= \int_0^1 \sqrt{1+4x}dx \\ &= \frac{1}{4}\int \sqrt{u}du \text{ (Using u-substitution)} \\ &= \frac{1}{6}(1+4x)^{3/2}|_0^1 \\ &= \frac{1}{6}(5^{3/2}-1) \end{aligned}$$

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Question 1

Use the disk method. Then

$$\begin{aligned} V &= \int_0^{\pi/2} \pi(\sqrt{\cos(x)})^2 dx \\ &= \int_0^{\pi/2} \pi\cos(x)dx \\ &= \pi\sin(x)|_0^{\pi/2} \\ &= \pi \end{aligned}$$

Question 2

$$\begin{aligned} L &= \int_1^5 \sqrt{1+f'(x)^2}dx \\ &= \int_1^5 \sqrt{10}dx \\ &= 4\sqrt{10} \end{aligned}$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_1^3 2\pi x(f(x)-g(x))dx \\ &= \int_1^3 2\pi x(2x-1)dx \\ &= \int_1^3 4\pi x^2 - 2\pi x dx \\ &= \frac{4}{3}\pi x^3 - \pi x^2|_1^3 \\ &= \frac{80\pi}{3} \end{aligned}$$

Question 4

A sphere can be formed by rotating the line $f(x) = \sqrt{r^2 - x^2}$ about the x-axis from $-r$ to r. Then

$$\begin{aligned} V &= \int_{-r}^{r} \pi(\sqrt{r^2-x^2})^2 dx \\ &= \int_{-r}^{r} \pi r^2 - \pi x^2 dx \\ &= \pi r^2 x - \frac{\pi}{3}x^3|_{-r}^{r} \\ &= (\pi r^3 - \frac{\pi}{3}r^3) - (-\pi r^3 - \frac{\pi}{3}(-r)^3) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

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Question 1

First note that $\cos(x) < 0$ for $x \in (\frac{\pi}{2}, \pi]$, and so $\sqrt{\cos(x)}$ is undefined there. We will instead compute the volume on the interval on $[0, \frac{\pi}{2}]$. Use the disk method. Then

$$\begin{aligned} V &= \int_0^{\pi/2} \pi(\sqrt{\cos(x)})^2 dx \\ &= \int_0^{\pi/2} \pi \cos(x) dx \\ &= \pi \sin(x)|_0^{\pi/2} \\ &= \pi \end{aligned}$$

Question 2

$$\begin{aligned} L &= \int_1^5 \sqrt{1+f'(x)^2}dx \\ &= \int_1^5 \sqrt{2}dx \\ &= 4\sqrt{2} \end{aligned}$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_1^2 2\pi x(f(x)-g(x))dx \\ &= \int_1^2 2\pi x(3x-(x-1))dx \\ &= \int_1^2 4\pi x^2 + 2\pi x dx \\ &= \frac{4}{3}\pi x^3 + \pi x^2|_1^2 \\ &= \frac{37\pi}{3} \end{aligned}$$

Question 4

A sphere can be formed by rotating the line $f(x) = \sqrt{r^2-x^2}$ about the x-axis from $-r$ to r. Then

$$\begin{aligned} V &= \int_{-r}^r \pi(\sqrt{r^2-x^2})^2 dx \\ &= \int_{-r}^r \pi r^2 - \pi x^2 dx \\ &= \pi r^2 x - \frac{\pi}{3}x^3|_{-r}^r \\ &= (\pi r^3 - \frac{\pi}{3}r^3) - (-\pi r^3 - \frac{\pi}{3}(-r)^3) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

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Question 1

Use the disk method. Then

$$\begin{aligned} V &= \int_0^{\pi/4} \pi(\sqrt{\sin(x)})^2 dx \\ &= \int_0^{\pi/4} \pi \sin(x) dx \\ &= -\pi \cos(x)|_0^{\pi/4} \\ &= \frac{2-\sqrt{2}}{2}\pi \end{aligned}$$

Question 2

$$\begin{aligned} L &= \int_1^5 \sqrt{1+f'(x)^2} dx \\ &= \int_1^5 \sqrt{2} dx \\ &= 4\sqrt{2} \end{aligned}$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_1^3 2\pi x(f(x)-g(x)) dx \\ &= \int_0^3 2\pi x(2x-x) dx \\ &= \int_0^3 2\pi x^2 dx \\ &= \frac{2}{3}\pi x^3|_0^3 \\ &= 18\pi \end{aligned}$$

Question 4

$$
\begin{aligned}
L &= \int_1^2 \sqrt{1+f'(x)^2}dx \\
&= \int_1^2 \sqrt{1+(3x^{1/2})^2}dx \\
&= \int_1^2 \sqrt{1+9x}dx \\
&= \frac{1}{9}\int \sqrt{u}du \text{ (Using u-substitution)} \\
&= \frac{2}{27}(1+9x)^{3/2}|_1^2 \\
&= \frac{2}{27}(19^{3/2}-10^{3/2})
\end{aligned}
$$

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Question 1

First note that $\cos(x) < 0$ for $x \in (\pi, \frac{3\pi}{2}]$, and so $\sqrt{\cos(x)}$ is undefined there. We will instead use the function $f(x) = \sqrt{|\cos(x)|}$. Use the disk method. Then

$$
\begin{aligned}
V &= \int_\pi^{3\pi/2} \pi(\sqrt{|\cos(x)|})^2 dx \\
&= \int_\pi^{3\pi/2} \pi|\cos(x)|dx \\
&= \int_\pi^{3\pi/2} \pi(-\cos(x))dx \\
&= -\pi\sin(x)|_\pi^{3\pi/2} \\
&= \pi
\end{aligned}
$$

Question 2

$$
\begin{aligned}
L &= \int_2^4 \sqrt{1+f'(x)^2}dx \\
&= \int_2^4 \sqrt{2}dx \\
&= 2\sqrt{2}
\end{aligned}
$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_2^3 2\pi x(f(x) - g(x))dx \\ &= \int_2^3 2\pi x(1 - (x-2))dx \\ &= \int_2^3 6\pi x - 2\pi x^2 dx \\ &= 3\pi x^2 - \frac{2}{3}\pi x^3|_2^3 \\ &= \frac{7\pi}{3} \end{aligned}$$

Question 4

A sphere can be formed by rotating the line $f(x) = \sqrt{r^2 - x^2}$ about the x-axis from $-r$ to r. Then

$$\begin{aligned} V &= \int_{-r}^{r} \pi(\sqrt{r^2 - x^2})^2 dx \\ &= \int_{-r}^{r} \pi r^2 - \pi x^2 dx \\ &= \pi r^2 x - \frac{\pi}{3}x^3|_{-r}^{r} \\ &= (\pi r^3 - \frac{\pi}{3}r^3) - (-\pi r^3 - \frac{\pi}{3}(-r)^3) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

1 PM

Question 1

First note that $\sin(x) < 0$ for $x \in (-\pi, 0)$, and so $\sqrt{\sin(x)}$ is undefined there. We will instead use the function $f(x) = \sqrt{|\sin(x)|}$. Use the disk method. Then

$$\begin{aligned} V &= \int_{-\pi}^{0} \pi(\sqrt{|\sin(x)|})^2 dx \\ &= \int_{-\pi}^{0} \pi|\sin(x)|dx \\ &= \int_{-\pi}^{0} \pi(-\sin(x))dx \\ &= \pi\cos(x)|_{-\pi}^{0} \\ &= 2\pi \end{aligned}$$

Question 2

$$\begin{aligned} L &= \int_1^5 \sqrt{1+f'(x)^2}dx \\ &= \int_1^5 \sqrt{10}dx \\ &= 4\sqrt{10} \end{aligned}$$

Question 3

Use the shell method

$$\begin{aligned} V &= \int_1^3 2\pi x(f(x)-g(x))dx \\ &= \int_1^3 2\pi x(3-(-2))dx \\ &= \int_1^3 10\pi x dx \\ &= 5\pi x^2|_1^3 \\ &= 40\pi \end{aligned}$$

Question 4

$$\begin{aligned} L &= \int_0^2 \sqrt{1+f'(x)^2}dx \\ &= \int_0^2 \sqrt{1+(\frac{3}{2}x^{1/2})^2}dx \\ &= \int_0^2 \sqrt{1+\frac{9}{4}x}dx \\ &= \frac{4}{9}\int \sqrt{u}du \text{ (Using u-substitution)} \\ &= \frac{8}{27}u^{3/2}|_*^* \\ &= \frac{8}{27}(1+\frac{9}{4}x)^{3/2}|_0^2 \\ &= \frac{8}{27}((\frac{11}{2})^{3/2}-1) \end{aligned}$$