8 AM

Question 1

$$\lim_{n\to\infty} \sin\left(\frac{2^n}{3^n}\right) = \sin\left(\lim_{n\to\infty}\left(\frac{2}{3}\right)^n\right) = \sin(0) = 0.$$

Question 2

Note that we know $\lim\limits_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)^{1/n} = \frac{\lim\limits_{n\to\infty} 1^{\frac{1}{n}}}{\lim\limits_{n\to\infty} n^{1/n}} = \frac{1}{1} = 1.$$

Question 3

The integral diverges to $-\infty$ because

$$\int_0^1 \frac{1}{x-1} = \lim_{A\to 1}\int_0^A \frac{1}{x-1} = \lim_{A\to 1} \ln|x-1|\,\Big|_0^A = \lim_{A\to 1} \ln|A-1| = -\infty$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \leq \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a,b]$. We have $f(x) = \sin(x)$, so $f''(x) = -\sin(x)$ and $|f''(x)| = |\sin(x)|$ has a maximum of 1 on $[\pi, 3\pi]$. Therefore the error is bounded by

$$E_n^T \leq \frac{(2\pi)^3}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \leq \frac{(2\pi)^3}{12n^2} \leq \frac{1}{10} \implies n \geq \sqrt{\frac{20\pi}{3}} \approx 14.4.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 15$.

9 AM

Question 1

Let $u = 3n$, and so as n goes to infinity, so does u. Then we have

$$\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^n = \lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{\frac{u}{3}} = \left(\lim_{n\to\infty}\left(1+\frac{1}{u}\right)^u\right)^{\frac{1}{3}} = e^{\frac{1}{3}}$$

since we know that $\lim\limits_{n\to\infty}(1+\frac{1}{u})^u = e$.

Question 2

Note that we know $\lim\limits_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty}(3n)^{1/n} = \lim_{n\to\infty} 3^{1/n}\cdot\lim_{n\to\infty} n^{1/n} = 1\cdot 1 = 1.$$

Question 3

The integral diverges to ∞ because

$$\int\limits_{-1}^{1}\frac{1}{x+1} = \lim_{A\to-1}\int\limits_{A}^{1}\frac{1}{x+1} = \lim_{A\to-1}\ln|x+1|\,|_A^1 = \lim_{A\to-1}\ln|2| - \ln|A+1| = \infty.$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \leq \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a,b]$. We have $f(x) = e^x$, so $f''(x) = e^x$ and $|f''(x)| = e^x$ has a maximum of e on $[0,1]$. Therefore the error is bounded by

$$E_n^T \leq \frac{e}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \leq \frac{e}{12n^2} \leq \frac{1}{10} \implies n \geq \sqrt{\frac{5e}{6}} \approx 1.4.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 2$.

10 AM

Question 1

Let $u = \frac{n}{2}$, and so as n goes to infinity, so does u. Then we have

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = \lim_{u\to\infty}\left(1+\frac{1}{u}\right)^{2u} = \left(\lim_{n\to\infty}\left(1+\frac{1}{u}\right)^u\right)^2 = e^2$$

since we know that $\lim\limits_{n\to\infty}(1+\frac{1}{u})^u = e$.

Question 2

Note that we know $\lim\limits_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty}(n)^{1/2n} = (\lim_{n\to\infty} n^{1/n})^{1/2} = 1^{1/2} = 1.$$

Question 3

The integral converges to 1 because

$$\int_1^\infty \frac{1}{x^2} = \lim_{A\to\infty}\int_1^A \frac{1}{x^2} = \lim_{A\to\infty}\frac{-1}{x}\,|_1^A = \lim_{A\to\infty} 1 - \frac{1}{A} = 1.$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \le \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a,b]$. We have $f(x) = \cos(x)$, so $f''(x) = -\cos(x)$ and $|f''(x)| = |\cos(x)|$ has a maximum of 1 on $[0, 4\pi]$. Therefore the error is bounded by

$$E_n^T \le \frac{(4\pi)^3}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \le \frac{(4\pi)^3}{12n^2} \le \frac{1}{10} \implies n \ge \sqrt{\frac{80\pi}{3}} \approx 28.8.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 29$.

11 AM

Question 1

Note that we know $\lim_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty} \cos(2\pi n^{1/n}) = \cos(\lim_{n\to\infty} 2\pi \cdot \lim_{n\to\infty} n^{1/n}) = \cos(2\pi \cdot 1) = \cos(2\pi) = 1.$$

Question 2

Note that we know $\lim_{n\to\infty} n^{1/n} = 1$. Let $u = 2n$, so u goes to ∞ as n goes to ∞. Now we have

$$\lim_{n\to\infty} \left(\frac{1}{2n}\right)^{\frac{1}{2n}} = \lim_{u\to\infty} \left(\frac{1}{u}\right)^{\frac{1}{u}} = \frac{\lim_{u\to\infty} 1}{\lim_{u\to\infty} u^{1/u}} = \frac{1}{1} = 1.$$

Question 3

The integral converges to 1 because

$$\int_1^\infty \frac{1}{x^2} = \lim_{A\to\infty} \int_1^A \frac{1}{x^2} = \lim_{A\to\infty} \frac{-1}{x} \big|_1^A = \lim_{A\to\infty} 1 - \frac{1}{A} = 1.$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \le \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a, b]$. We have $f(x) = x^2$, so $f''(x) = 2$ and $|f''(x)| = |2|$ has a maximum of 2 on $[1, 2]$. Therefore the error is bounded by

$$E_n^T \le \frac{2}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \le \frac{2}{12n^2} \le \frac{1}{10} \implies n \ge \sqrt{\frac{5}{3}}.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 2$.

12 PM

Question 1

Since $-1 \leq \sin(2x) \leq 1$ we have

$$\frac{-1}{n} \leq \frac{\sin(2n)}{n} \leq \frac{1}{n}.$$

Since both $\frac{1}{n}$ and $\frac{-1}{n}$ converge to 0 as $n \to \infty$, by the squeeze theorem we have

$$\lim_{n\to\infty} \frac{\sin(2n)}{n} = 0.$$

Question 2

Note that we know $\lim\limits_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty}(n^2)^{\frac{1}{2n}} = (\lim_{n\to\infty} n^{1/n})^{1/2} \cdot (\lim_{n\to\infty} n^{1/n})^{1/2} = 1 \cdot 1 = 1.$$

Question 3

The integral diverges to ∞ because

$$\int_1^\infty \frac{1}{x} = \lim_{A\to\infty} \int_1^A \frac{1}{x} = \lim_{A\to\infty} \ln|x| \,|_1^A = \lim_{A\to\infty} \ln(A) = \infty.$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \leq \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a,b]$. We have $f(x) = e^{-x}$, so $f''(x) = e^{-x}$ and $|f''(x)| = |e^{-x}|$ has a maximum of 1 on $[0,1]$. Therefore the error is bounded by

$$E_n^T \leq \frac{1}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \leq \frac{1}{12n^2} \leq \frac{1}{10} \implies n \geq \sqrt{\frac{5}{6}}.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 1$.

1 PM

Question 1

$$\lim_{n\to\infty} \frac{n^2+3n+7}{-n^3+n^2-n+1} = \lim_{n\to\infty} \frac{n^2(1+\frac{3}{n}+\frac{7}{n^2})}{n^3(-1+\frac{1}{n}-\frac{1}{n^2}+\frac{1}{n^3})} = \lim_{n\to\infty} \frac{-1}{n} = 0.$$

Question 2

Note that we know $\lim\limits_{n\to\infty} n^{1/n} = 1$. Now we have

$$\lim_{n\to\infty} (n^3)^{1/n} = \lim_{n\to\infty} n^{1/n} \cdot \lim_{n\to\infty} n^{1/n} \cdot \lim_{n\to\infty} n^{1/n} = 1 \cdot 1 \cdot 1 = 1.$$

Question 3

The integral diverges to ∞ because

$$\int_1^\infty \frac{1}{x} = \lim_{A\to\infty} \int_1^A \frac{1}{x} = \lim_{A\to\infty} \ln|x| \,|_1^A = \lim_{A\to\infty} \ln(A) = \infty.$$

Question 4

The error for using the Trapezoid Rule with n subintervals is denoted E_n^T and we have the inequality

$$E_n^T \leq \frac{K_T(b-a)^3}{12n^2}$$

where K_T denotes the maximum of the absolute value of the second derivative of f on the interval $[a,b]$. We have $f(x) = e^x$, so $f''(x) = e^x$ and $|f''(x)| = e^x$ has a maximum of e^2 on $[0,2]$. Therefore the error is bounded by

$$E_n^T \leq \frac{e^2}{12n^2}.$$

We want this error to be bounded above by $\frac{1}{10}$, so we have

$$E_n^T \leq \frac{e^2}{12n^2} \leq \frac{1}{10} \implies n \geq \sqrt{\frac{5e^2}{6}} \approx 2.5.$$

We know n must be an integer, so the smallest value of n for which this inequality holds is $n = 3$.