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Question 1

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{3^{n+1}(n+1)!}{(2n+3)!} \cdot \frac{(2n+1)!}{3^n n!} = \lim_{n\to\infty} \frac{3n}{(2n+3)(2n+2)} = 0 < 1$$

so the series converges.

Question 2

We can rewrite this series as the geometric series $\sum_{n=1}^{\infty} (2x)^n$ which converges when $|2x| < 1$ or when $|x| < \frac{1}{2}$. Thus the radius of convergence is $\frac{1}{2}$.

Question 3

This is just the sum of a geometric series starting at $n = 2$ with $r = -1/3$, so the sum is

$$\frac{(-\frac{1}{3})^2}{1 + \frac{1}{3}} = \frac{1}{12}.$$

Question 4

Applying the ratio test we have

$$\lim_{n\to\infty} \frac{x^{n+2}}{(n+2)!} \cdot \frac{(n+1)!}{x^{n+1}} = \lim_{n\to\infty} \frac{x}{n+2} = 0 < 1.$$

Since this holds for all values of x, the radius of convergence is infinite, and thus the interval of convergence is $(-\infty, \infty)$.

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Question 1

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{2^{n+1}(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{2^n n!} = \lim_{n\to\infty} \frac{2(n+1)}{(2n+2)(2n+1)} = 0 < 1,$$

so the series converges.

Question 2

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{2^{n+1}x^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n x^n} = \lim_{n\to\infty} \frac{2x}{n+1} = 0 < 1.$$

Since this holds for all values of x, the radius of convergence is infinite.

Question 3

First note that by plugging 1 into the Taylor series for e^x, we have

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e.$$

By subtracting the first term of the series and multiplying by 2 we get

$$\sum_{n=1}^{\infty} \frac{2}{n!} = 2(e-1).$$

Question 4

We can rewrite this series as the geometric series $\sum_{n=1}^{\infty}(2x)^n$ which converges when $|2x| < 1$ or when $|x| < \frac{1}{2}$. When $x = \frac{1}{2}$, we have $\sum_{n=1}^{\infty} 1$ which clearly does not converge, and when $x = -\frac{1}{2}$ we have $\sum_{n=1}^{\infty}(-1)^n$ which also clearly does not converge. Therefore the interval of convergence is $(-\frac{1}{2}, \frac{1}{2})$.

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Question 1

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{(2n+3)!}{(n+1)^{n+1}} \cdot \frac{n^n}{(2n+1)!} = \lim_{n\to\infty} \frac{(2n+3)(2(n+1))n^n}{(n+1)(n+1)^n} = \lim_{n\to\infty} 4n+6 \cdot (\frac{n}{n+1})^n = \lim_{n\to\infty} \frac{4n+6}{e} = \infty.$$

Therefore the series diverges.

Question 2

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{(n+1)!x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n!x^n} = \lim_{n\to\infty} \frac{nx}{2} = \infty$$

for all values of x except for $x = 0$. Therefore the radius of convergence is 0.

Question 3

Note that we can rewrite the series as a two times a geometric series. Evaluating, we get

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = 2\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = 2 \cdot \frac{\frac{2}{3}}{1-\frac{2}{3}} = 4.$$

Question 4

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{(n+1)x^{n+1}}{nx^n} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \lim_{n\to\infty} x = x$$

which is less than one exactly when $x < 1$. Therefore the radius of convergence is 1, and all that remains is to check $x = 1$ and $x = -1$. At $x = 1$ we get the series $\sum_{n=2}^{\infty} n$ which clearly diverges, and similarly at $x = -1$ we get the series $\sum_{n=2}^{\infty} (-1)^n n$ which also clearly diverges since the terms do not go to 0. Therefore the interval of convergence is $(-1, 1)$.

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Question 1

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{(2n+2)!}{2^{n+1}(n+1)!} \cdot \frac{2^n n!}{(2n)!} = \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{2(n+1)} = \infty,$$

so the series diverges.

Question 2

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{3^{n+1}x^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{3^n x^n} = \lim_{n\to\infty} \frac{3x}{(2n+2)(2n+1)} = 0 < 1$$

for all values of x, so the radius of convergence is infinite.

Question 3

We can index the series to start at $n = 0$ instead of $n = 1$, and rewriting it we get

$$\sum_{n=1}^{\infty} \frac{3}{(n-1)!} = \sum_{n=0}^{\infty} \frac{3}{n!} = 3 \cdot \sum_{n=0}^{\infty} \frac{1}{n!} = 3e.$$

The last equality holds because $\sum_{n=0}^{\infty} \frac{1}{n!}$ is just the Taylor series for e^x evaluated at $x = 1$.

Question 4

We can rewrite this series as the geometric series $\sum_{n=3}^{\infty} (3x)^n$ which converges when $|3x| < 1$ or when $|x| < \frac{1}{3}$. When $x = \frac{1}{3}$, we have $\sum_{n=1}^{\infty} 1$ which clearly does not converge, and when

$x = -\frac{1}{3}$ we have $\sum_{n=1}^{\infty}(-1)^n$ which also clearly does not converge. Therefore the interval of convergence is $(-\frac{1}{3}, \frac{1}{3})$.

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Question 1

Note that $\ln(x)$ is a strictly increasing function, so

$$\lim_{n\to\infty} \ln(1+n) = \infty.$$

Since the terms of the series do not go to 0, the series diverges.

Question 2

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{2^{n+1}x^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{2^n x^n} = \lim_{n\to\infty} \frac{2x}{n+2} = 0 < 1.$$

Since this holds for all values of x, the radius of convergence is infinite.

Question 3

We can rewrite the series as

$$\sum_{n=1}^{\infty} \frac{2}{3^n} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{3^n} = 2 \cdot \frac{\frac{1}{3}}{1-\frac{1}{3}} = 1$$

where the last equality holds because $\sum_{n=1}^{\infty} \frac{1}{3^n}$ is a geometric series.

Question 4

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{x^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{x^n} = \lim_{n\to\infty} \frac{x}{(2n+2)(2n+1)} = 0 < 1.$$

Since this holds for all values of x, the radius of convergence is infinite. Therefore the interval of convergence is $(-\infty, \infty)$.

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Question 1

This series will diverge because the terms do not go to 0. For example,

$$\lim_{n\to\infty} \frac{n^n}{n!} \geq n$$

for all n, and $\lim\limits_{n\to\infty} n = \infty$ so $\lim\limits_{n\to\infty} \frac{n^n}{n!} = \infty$ as well.

Question 2

By factoring out a 3 from the numerator we can rewrite the series as a geometric series. This gives us

$$\sum_{n=0}^{\infty} \frac{3^{n+1}x^n}{2^n} = 3 \cdot \sum_{n=0}^{\infty} \left(\frac{3x}{2}\right)^n$$

which converges only when $\frac{3x}{2} < 1$ which happens when $x < \frac{2}{3}$. Therefore the radius of convergence is $\frac{2}{3}$.

Question 3

If we factor out 5 from the numerator, then we get five times the Taylor series for e^x evaluated at 1. However since the sum starts at $n = 2$, we have to subtract the first two terms. Therefore we have

$$\sum_{n=2}^{\infty} \frac{5}{n!} = 5 \cdot \sum_{n=2}^{\infty} \frac{1}{n!} = 5(e - 1 - 1) = 5(e - 2).$$

Question 4

Applying the ratio test, we have

$$\lim_{n\to\infty} \frac{(n+1)^2 x^{n+1}}{n^2 x^n} = \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \cdot \lim_{n\to\infty} x = x$$

which is less than one exactly when $x < 1$. Therefore the radius of convergence is 1, and all that remains is to check $x = 1$ and $x = -1$. At $x = 1$ we get the series $\sum\limits_{n=2}^{\infty} n^2$ which clearly diverges, and similarly at $x = -1$ we get the series $\sum\limits_{n=2}^{\infty} (-1)^n n^2$ which also clearly diverges since the terms do not go to 0. Therefore the interval of convergence is $(-1, 1)$.