MATH 141 Quiz 6, 8am

1.) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^{2n}$ Using the ratio test to find the radius of convergence we take $lim_{n\to\infty} \frac{\frac{(n+1)!x^{2n+2}}{(n+1)^{n+1}}}{\frac{n!x^{2n}}{n^n}} =$

$lim_{n\to\infty} \frac{(n+1)n!x^{2n}x^2n^n}{(n+1)^n(n+1)n!x^{2n}} = lim_{n\to\infty} x^2(\frac{n}{n+1})^n = lim_{n\to\infty} x^2(1+\frac{1}{n})^{-n} = x^2e^{-1}$ Thus the radius of convergence is $e^{1/2}$.

2.) $\sum_{n=0}^{\infty} \frac{2}{n!} = 2\sum_{n=0}^{\infty} \frac{1}{n!} = 2\sum_{n=0}^{\infty} \frac{1^n}{n!} = 2e^1$

3.) We know the Taylor series about 0, $sin(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} t^{2n+1}$ now performing a simple change of varialbes $t = 3x$ we have $sin(3x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(3x)^{2n+1}$

4.) $\frac{2-3i}{3+4i} = \frac{2-3i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{6-12-9i-8i}{3^2+4^2} = \frac{-6}{25} - i\frac{17}{25}$

MATH 141 Quiz 6, 9am

1.) $\sum_{n=1}^{\infty} \frac{n^n}{n!}x^{2n}$ Using the ratio test to find the radius of convergence we take $lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}x^{2n+2}}{(n+1)!}}{\frac{n^n x^{2n}}{n!}} =$ $lim_{n\to\infty} \frac{(n+1)^n(n+1)n!x^{2n}x^2}{(n+1)n!x^{2n}n^n} = lim_{n\to\infty} x^2(\frac{n+1}{n})^n = lim_{n\to\infty} x^2(1+\frac{1}{n})^n = x^2e^1$ Thus the radius of convergence is $e^{-1/2}$.

2.) We know that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$. Thus $\sum_{n=0}^{\infty} (\frac{-1}{2})^n = \frac{1}{1-(\frac{-1}{2})} = \frac{2}{3}$.

3.) We know the Taylor series about 0, $sin(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}t^{2n+1}$ now performing a simple change of varialbes $t = x - 1$ we have $sin(x-1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}(x-1)^{2n+1}$.

4.)$\frac{2+3i}{3-4i} = \frac{2+3i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{6-12+9i+8i}{3^2+4^2} = \frac{-6}{25} + i\frac{17}{25}$

MATH 141 Quiz 6, 10am

1.) $\sum_{n=1}^{\infty} \frac{2^n}{n^n} x^n$ Using the root test to find the radius of convergence we take $lim_{n\to\infty}((\frac{2x}{n})^n)^{\frac{1}{n}} = lim_{n\to\infty} \frac{2x}{n} = 0$ thus the radius of convergence is ∞.

2.) $\int [\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n] dx = \sum_{n=0}^{\infty} [\int \frac{(-1)^n}{n!} x^n dx] = [\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{n+1} x^{n+1}] + C = [\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!} x^{n+1}] + C$

3.) We know the Taylor series about 0, $e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!}$ now performing a simple change of varialbes $t = x^2$ we have $e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

4.) $|4 - 3i| = \sqrt{(4-3i)(4+3i)} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

MATH 141 Quiz 6, 11am

1.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^n} x^n$ Using the generalized root test to find the radius of convergence we take $lim_{n\to\infty}(|(\frac{-x}{n})^n|)^{\frac{1}{n}} = lim_{n\to\infty}\frac{x}{n} = 0$ thus the radius of convergence is ∞. Making the interval of convergence $(-\infty, \infty)$.

2.) $\frac{d}{dx}[\sum_{n=1}^{\infty} x^n] = \sum_{n=1}^{\infty} \frac{d}{dx}[x^n] = \sum_{n=1}^{\infty} nx^{(n-1)}$

3.) We know the Taylor series about 0, $cos(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} t^{2n}$ now performing a simple change of varialbes $t = 2x$ we have $cos(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(2x)^{2n}$

4.) $|3 - 4i| = \sqrt{(3-4i)(3+4i)} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$