MATH 416, Spring 10, Practice Problems

1. Write explicitely the matrix of a 4×4 DFT. Apply it to a vector $(0, 1, 0, -1)$.

2. Find the Lagrange polynomial through points $(1, 2)$, $(2, 5)$, $(3, 4)$.

3. Suppose that $f(x) = mx$ for some constant m. Show that for any sampling of f, the piecewise linear approximation exactly equals f.

4. Let $\{h(k) : k \in \mathbb{Z}\}$ be a CQF sequence. Show that so is the sequence defined by

$$\forall\, k \in \mathbb{Z}, \quad g(k) = (-1)^k \overline{h(2 - 1 - k)}.$$

5. Show that the set of functions $\{\sqrt{2} \sin(\pi n t) : n = 1, 2, 3, \ldots\}$ is orthonormal with respect to the real inner product: $\langle f, g\rangle = \int_0^1 f(t)g(t)\, dt$.

6. Show that the set of vectors $\omega_n \in \mathbb{C}^N, n = 0, \ldots, N-1$, where $\omega_n(k) = 1/\sqrt{N} e^{2\pi i n k/N}$, is an orthonormal basis for $\mathbb{C}^N$ with respect to the complex inner product: $\langle v, w\rangle = \sum_{k=0}^{N-1} \overline{v(k)} w(k)$.

7. Prove that the $N \times N$ discrete Hartley transform matrix is symmetric and unitary.

8. Find the expansion in Chebyshev polynomials $T_0(x), T_1(x), T_2(x)$ of the function $f(x) = 1 + x^2$ dened for $x \in [-1, 1]$.

9. Compute the sine-cosine Fourier series of the 1-periodic function $f(x) = \cos^2(2\pi x)$.

10. Compute the complex exponential Fourier series of the 1-periodic function $\sin(2\pi k t - d)$, where d is a constant real number, and k is an integer.