MATH 401, HW 7, FALL 2015

1. (**12 points**) Write your own Matlab code that solves a system of equations of the form $LU(x) = b$, where $b \in \mathbb{R}^n$ is a given vector of constants, U is an upper triangular $n \times n$ matrix, and L is a lower triangular $n \times n$ matrix. Apply this algorithm to a problem with $n = 10$.

Please make sure that your code takes advantage of this specific structure of the equation, and that it DOES NOT multiply LU into a single matrix.

Note that no inversion is needed for this problem. Note also that this problem is NOT about doing LU factorization. Here, you simply assume that a factored matrix LU is given to you.

2) (**5 points**) Find the computational complexity of the algorithm you designed for Problem 1, using the "big oh" notation, and assuming that the LU factored matrix is given. Compare it to the computational complexity of the standard Gauss-Jordan scheme for solving a system of n equations with n unknowns, and draw conclusions.

3) (**8 points**) Given matrix

$$A = \begin{pmatrix} 10^{-20} & 1 \\ 1 & 1 \end{pmatrix},$$

note that its LU factorization is given by

$$L = \begin{pmatrix} 1 & 0 \\ 10^{20} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 10^{-20} & 1 \\ 0 & 1 - 10^{20} \end{pmatrix}.$$

Assume the following representation format for real numbers: each real number which can be written as $a.bcdefg \ldots \times 10^N$, where $a, b, c, d \ldots \in \{0, 1, 2, 3, \ldots, 9\}$ and $N \in \mathbb{Z}$, is represented as $a.bc \times 10^N$. For example, 10^{-20} is represented as 1.00×10^{-20}, and π is represented as 3.14×10^0. For arithmetic operations assume that they can be performed exactly and then the result is rounded to fit the new representation scheme. For example, $\pi \times 10^{-20}$ is represented by 3.14×10^{-20}, and $e \times 10^{20}$ is represented by 2.72×10^{20}.

In this new representation scheme compute the product of L and U, and compare it to A. Estimate the difference. Propose a remedy for the LU factorization which will eliminate the observed difference.