MATH 630, Spring 2007, MIDTERM

1) Let $\{n_k : k = 1, \ldots\} \subseteq \mathbb{N}$ be a subsequence of natural numbers. Show that

$$m(\{x : \liminf_{k\to\infty} \sin(n_k x) > 0\}) = 0.$$

2) Let $k \in L^1_m(\mathbb{R})$ be a non-negative function with $\int_{\mathbb{R}} k = 1$. Let $g : \mathbb{R} \to \mathbb{R}$ be a bounded and continuous function. Show that

$$\lim_{n\to\infty} \int_{\mathbb{R}} nk(nx)g(x)\, dx = g(0).$$

3) Let $(X, \mathcal{Z}, \mu)$ be a measure space and let $\{A_n : n = 1, \ldots\} \subseteq \mathcal{Z}$. Assume that $A_n \subseteq A_{n+1}$, $n \in \mathbb{N}$, ane let $A = \bigcup_{n=1}^{\infty} A_n$. Prove that $\mu(A) = \lim_{n\to\infty} \mu(A_n)$..

4) Let $f : \mathbb{R} \to \mathbb{R}^+$, $f \in L^1_m(\mathbb{R})$. Let $A_\alpha = \{x \in \mathbb{R} : f(x) > \alpha\}$, $\alpha > 0$. Prove that

$$m(A_\alpha) \leq \frac{1}{\alpha} \int_{\mathbb{R}} f.$$

5) Problem 3.13

6) Problem 3.22