SOLUTION TO PROBLEM 4 ON MIDTERM 2

You should start by simplifying the trigonometric functions, like this:

$$\int \sin^2(x) \cos^3(x) \sec^2(x) \, dx = \int \sin^2(x) \cos^3(x) \frac{1}{\cos^2(x)} \, dx = \int \sin^2(x) \cos(x) \, dx.$$

Next, you should observe that this is a trigonometric integral that can be solved by substituting $y = \sin(x)$, with $dy = \cos(x) \, dx$. This way the integral becomes:

$$\int y^2 \, dy = \frac{1}{3} y^3 + C.$$

Substituting back $\sin(x) = y$, we get

$$\int \sin^2(x) \cos^3(x) \sec^2(x) \, dx = \frac{1}{3} \sin^3(x) + C.$$

Please remember that you can check yourself if your antiderivative is correct by differentiating it and comparing to the function under integral.