1. Find the limit

$$\lim_{n\to\infty} \sqrt[n]{\ln(n)}$$

There were two ways to approach this problem: (1) using the squeeze theorem and (2) using l'Hopitals rule on the associated function.

(1) The Squeeze Theorem:

Note that $\sqrt[n]{x} \geq 1$ for $x \geq 1$. So for $n > 2$, $\sqrt[n]{\ln(n)} \geq 1$. Also, $\ln(n) \leq n$ for all positive integers and so $\sqrt[n]{\ln(n)} \leq \sqrt[n]{n}$. Summarizing, we have for $n > 2$:

$$1 \leq \sqrt[n]{\ln(n)} \leq \sqrt[n]{n}$$

We know that $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$ (this is computed as an example in the text.) Then by the squeeze theorem,

$$\lim_{n\to\infty} \sqrt[n]{\ln(n)} = 1 \ .$$

(2) l'Hopital's Rule:

Consider the function $f(x) = \sqrt[x]{\ln(x)}$. Rewrite the function as $f(x) = \exp[\frac{1}{x}\ln(\ln x)]$. We want to compute $\lim\limits_{x\to\infty} f(x)$:

$$\begin{aligned}
\lim_{x\to\infty} f(x) &= \exp[\lim_{x\to\infty} \tfrac{1}{x}\ln(\ln x)] && \text{(by continuity of } \exp[\,]) \\
&= \exp[\lim_{x\to\infty} \tfrac{1}{x\ln x}] && \text{(by l'Hopital)} \\
&= \exp[0] \\
&= 1
\end{aligned}$$

Therefore

$$\lim_{n\to\infty} \sqrt[n]{\ln(n)} = 1 \ .$$

The most common mistakes were to argue in someway that $\lim\limits_{n\to\infty} \sqrt[n]{\ln(n)} = \infty^0 = 1$ or to say that $\lim\limits_{n\to\infty} \sqrt[n]{\ln(n)} = \lim\limits_{n\to\infty} \frac{\ln n}{n} = 0$. The former is a nonsensical statement. The same argument would say that $n = \sqrt[n]{n^n} \to 1$ as $n \to \infty$, which is clearly false. The latter is a misunderstanding of the rules for logarithms: $\frac{\ln n}{n} = \ln(\sqrt[n]{n})$ not $\sqrt[n]{\ln n}$.

Because the method was an essential part of the problem, for the most part no points were given if neither (1) nor (2) were employed.