

The ODE is given by

$$\frac{dy}{dx} = y + x^2$$

It is easy to see this is not linear, and we can put it in the standard form

$$\frac{dy}{dx} - y = x^2$$

so

$$P(x) = -1, Q(x) = x^2, S(x) = -x$$

It follows that the general solution is

$$y = e^x \int e^{-x} x^2 dx$$

The above integral can be done using integration by parts twice (or tabular integration). The table for the first integration by parts is as follows:

$$\begin{array}{ll} u = x^2 & dv = e^{-x} \\ du = 2x & v = -e^{-x} \end{array}$$

we get

$$\int e^{-x} x^2 dx = -e^{-x} x^2 + 2 \int x e^{-x} dx$$

We do a second integration by parts to obtain

$$\int e^{-x} x^2 dx = -e^{-x} x^2 - 2x e^{-x} - 2 + C$$

where C is a constant. Applying the above formula for the general solution y , we get

$$y = e^x (-e^{-x} x^2 - 2x e^{-x} - 2 + C) = -x^2 - 2x - 2 + C e^x$$

We can verify this is the solution by plugging it into the differential equation.

The general form of the differential equation was something you can memorize, whereas solving the integral took a little work. I awarded 15 points to the general solution form and 10 points to the integration (5 to each integration by parts). Flagrant violations of algebraic or integration rules resulted in massive point losses. (For example dividing the left hand side of the equation by something and the right hand side by something else, or declaring $\int f * g = \int f + \int g$). For this reason, students who thought the equation was separable or who didn't apply integration by parts did poorly.

I was understanding of errors that changed the problem only slightly (such as thinking $P(x) = 1$ rather than -1). But errors that made the problem much easier resulted in more point losses. Also, forgetting the constant of integration was a more minor offense than failing to distribute the e^x properly at the end. This is because $-x^2 - 2x - 2 + C$ doesn't solve the differential equation unless $C = 0$ (as you can check), but $-x^2 - 2x - 2$ does.