

MATH 141, FALL 2013, MIDTERM 2

Problem 2. Let  $R$  be the region bounded by the y-axis and by the right half of the circle centered at the origin with radius equal to 1. Sketch the region  $R$  and find the coordinates of its center of gravity.

Total points: 25.

1. (6 pt.) Sketch of the region.
2. (7 pt.) The moment about the x-axis is given by:

$$M_x = \int_{-1}^1 yf(y)dy = \int_{-1}^1 y\sqrt{1-y^2}dy = 0$$

because the integrand is an odd function.

3. (7 pt.) The moment about the y-axis is given by:

$$\begin{aligned} M_y &= \frac{1}{2} \int_{-1}^1 [f(y)]^2 dy = \frac{1}{2} \int_{-1}^1 (1-y^2) dy = \int_0^1 (1-y^2) dy \\ &= \left[ y - \frac{y^3}{3} \right]_0^1 = \frac{2}{3} \end{aligned}$$

4. (3 pt.) The area of  $R$  is:

$$A = \int_{-1}^1 \sqrt{1-y^2} dy = \frac{\pi}{2}$$

5. (2 pt.) Then the coordinates of the center of gravity are:

$$\begin{aligned} \bar{x} &= \frac{M_y}{A} = \frac{4}{3\pi} \\ \bar{y} &= \frac{M_x}{A} = 0 \end{aligned}$$