Math141 Exam 4 Problem 2

October 2014

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n} x^{2n}$$

Solution (Ratio Test) The test is worth 15 points.

$$\lim_{n \to \infty} \left| \frac{\frac{(n+1)^2 x^{2n+2}}{3^{n+1}}}{\frac{n^2 x^{2n}}{3^n}} \right|$$

$$\lim_{n \to \infty} \left| \frac{(n+1)^2 x^2}{3n} \right|$$

$$|x^2/3| < 1$$

$$-\sqrt{3} < x < \sqrt{3}$$

(8 points) Now check the endpoints, $x = \sqrt{3}$, $x = -\sqrt{3}$.

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n} (\sqrt{3})^{2n} = \sum_{n=1}^{\infty} \frac{n^2}{1}$$

This diverges by Nth term divergence test.

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n} (-\sqrt{3})^{2n} = \sum_{n=1}^{\infty} \frac{n^2}{1}$$

This diverges by Nth term divergence test.

(2 points) The interval of convergence is

$$(-\sqrt{3}, \sqrt{3})$$

Solution (Root Test) The test is worth 15 points.

$$\lim_{n\to\infty}\left|\left(\frac{(n)^2 x^{2n}}{3^n}\right)^{1/n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n)^{2/n} x^2}{3}\right|$$

$$|x^2/3| < 1$$

$$-\sqrt{3} < x < \sqrt{3}$$

(8 points) Now check the endpoints, $x = \sqrt{3}$, $x = -\sqrt{3}$.

$$\sum_{n=1}^{\infty}\frac{n^2}{3^n}(\sqrt{3})^{2n} = \sum_{n=1}^{\infty}\frac{n^2}{1}$$

This diverges by Nth term divergence test.

$$\sum_{n=1}^{\infty}\frac{n^2}{3^n}(-\sqrt{3})^{2n} = \sum_{n=1}^{\infty}\frac{n^2}{1}$$

This diverges by Nth term divergence test.
(2 points) The interval of convergence is

$$(-\sqrt{3}, \sqrt{3})$$

The series is absolutely convergent on the interval of convergence.