Solution of Problem 3

First Method

$$\begin{aligned}
\int \frac{ln(x)}{x(1+ln^4(x))}\,dx &= \int \frac{u}{1+u^4}\,du, \text{ where } u = lnx\,,\ du = \frac{1}{x}\,dx \qquad (7 \text{ points}) \\
&= \frac{1}{2}\int \frac{1}{1+w^2}\,dw, \text{ where } w = u^2\,,\ dw = 2u\,du \quad (5 \text{ points}) \\
&= \frac{1}{2}\,tan^{-1}(w) + C \\
&= \frac{1}{2}\,tan^{-1}(ln^2(x)) + C \qquad (8 \text{ points})
\end{aligned}$$

Second Method

$$\begin{aligned}
\int \frac{ln(x)}{x(1+ln^4(x))}\,dx &= \frac{1}{2}\int \frac{1}{1+u^2}\,du, \text{ where } u = ln^2(x)\,,\ du = \frac{2\,lnx}{x}\,dx \qquad (12 \text{ points}) \\
&= \frac{1}{2}\,tan^{-1}(u) + C \\
&= \frac{1}{2}\,tan^{-1}(ln^2(x)) + C \qquad (8 \text{ points})
\end{aligned}$$