We want to know whether

$$\int_1^2 \frac{1}{x\sqrt{\ln(x)}}dx$$

converges, and if it does, we want to know its value.

The integrand is undefined at 1 since $\ln(1) = 0$. Thus the improper integral is by definition

$$\int_1^2 \frac{1}{x\sqrt{\ln(x)}}dx = \lim_{b\to 1^+} \int_b^2 \frac{1}{x\sqrt{\ln(x)}}dx$$

The integral is a simple u-substitution with $u = \ln(x)$ so $du = dx/x$ and

$$\int_{\ln(b)}^{\ln(2)} \frac{du}{\sqrt{u}} = 2\sqrt{u}\Big|_{\ln(b)}^{\ln(2)} = 2\sqrt{\ln(2)} - 2\sqrt{\ln(b)}$$

Now as $b \to 1^+$, we have

$$\lim_{b\to 1^+} \int_b^2 \frac{1}{x\sqrt{\ln(x)}}dx = 2\sqrt{\ln(2)} - 2\sqrt{\ln(1)} = 2\sqrt{\ln(2)}$$

Since the limit is a finite number, the improper integral converges to $2\sqrt{\ln(2)}$.

Most students did well on this problem. The grading was as follows: 5 points for recognizing why the integral is improper and rewriting it as a limit, 12 points for doing the integral correctly, 5 for calculating the limit properly, and 3 for stating the conclusion.

Some people correctly used integration by parts to solve the integral and were awarded full credit if they did it correctly.

Students lost a lot of points if they demonstrated they didn't understand the basic material. For example, many students did not understand how to combine logs, how to integrate, or how to differentiate, or made unjustified and magical algebraic manipulations without showing their work. On the other hand, simple transcription errors, and sign errors, did not result in much point loss unless they made the problem trivial.