Solution to #3, Test #4

December 8, 2011

#3. (Find the Taylor series expansion of f(x) = x/(1+x) around 0.)

Proof. f(x) = $\frac{x}{1+x}$ = 1 - $\frac{1}{1+x}$ f(0) = 0
f'(x) = $\frac{1}{(1+x)^2}$ f'(0) = 1
f"(x) = $\frac{-2}{(1+x)^3}$ f"(0) = -2
$f^{(n)}(x) = \frac{(-1)^n n!}{(1+x)^{n+1}}$ $f^{(n)}(0) = (-1)^{n+1}n!$

General form: f(x) = $\sum_{n=1}^{\infty}(-1)^{n+1}x^n$
Alternate Solution:
f(x) = $\frac{x}{1+x} = x\frac{1}{1-(-x)}$
Since $\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n$, it follows that
f(x) = x$\sum_{n=0}^{\infty}(-1)^n x^n = \sum_{n=0}^{\infty}(-1)^n x^{n+1}$

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Method 1:
6 points for the first few derivatives correctly computed and evaluated at 0
4 points for correct generalization of $f^n(0)$
5 points for correct coefficients in series
3 points for correct Taylor series form
2 points for correct starting index

Method 2:
5 points for correct formula for 1/(1-x)
5 points for formatting f(x) correctly to use the 1/(1-x) formula
5 points for correct series for 1/(1+x)
2 points for multiplying through by x
3 ponits for correct response