Math 141 Midterm 3 Question 3 Solution

December 3, 2015

Question: Find power series expansion of $f(x) = \frac{e^x - 1 - x}{x^2}$. (You may use Taylor series expansions already obtained in the textbook.) Then find its radius of convergence.

Solution:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \text{5 pts}$$

so, since 1 and x are the first two terms of this Taylor series,

$$e^x - 1 - x = \sum_{n=0}^{\infty} \frac{x^n}{n!} - 1 - x = \sum_{n=2}^{\infty} \frac{x^n}{n!} \qquad \text{3 pts}$$

$$\frac{e^x - 1 - x}{x^2} = \sum_{n=2}^{\infty} \frac{x^{n-2}}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{(n+2)!}. \qquad \text{2 pts}$$

Now we can find the radius using the (generalized) root test.

$$\lim_{n\to\infty} \left| \frac{x^{n+1} \cdot (n+2)!}{(n+3)! \cdot x^n} \right| \qquad \text{5 pts}$$

$$= \lim_{n\to\infty} \left| \frac{x}{n+2} \right|$$

$$= |x| \lim_{n\to\infty} \frac{1}{n+2} = 0 < 1 \qquad \text{3 pts}$$

Thus

$$R = \infty \qquad \text{2 pts}$$