Exam 1 Problem 4 Solution

Take the derivative (using the product rule)

$$f'(x) = \frac{(1+e^x)e^x - e^x e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} \qquad \text{(4 points)}$$

Notice that $e^x > 0$ for $x \in (-\infty, \infty)$. Hence

$$f'(x) > 0 \qquad \text{(4 points)}$$

Since f is increasing, it has an inverse on

$$(-\infty, \infty) \qquad \text{(3 points)}$$

The domain of f is $(-\infty, \infty)$, so the range of f^{-1} is also

$$(-\infty, \infty) \qquad \text{(3 points)}$$

To find $(f^{-1})'(\frac{1}{2})$, use the formula

$$(f^{-1})'(c) = \frac{1}{f'(a)} \qquad \text{(4 points)}$$

To find a, set $f(a) = \frac{1}{2}$.

$$\frac{e^a}{1+e^a} = \frac{1}{2} \Longrightarrow 2e^a = 1 + e^a \Longrightarrow e^a = 1 \Longrightarrow a = 0 \qquad \text{(4 points)}$$

Therefore

$$(f^{-1})'(\frac{1}{2}) = \frac{1}{f'(0)} = \frac{1}{\frac{e^0}{(1+e^0)^2}} = \frac{1}{\frac{1}{4}} = 4 \qquad \text{(3 points)}$$