Midterm 2

④ $\lim_{x\to\infty} \dfrac{\log_2(\log_3 x)}{\log_5(x)} = \lim_{x\to\infty} \dfrac{\ln\left(\frac{\ln x}{\ln 3}\right)/\ln 2}{\ln x/\ln 5}$

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\left[\dfrac{\ln\left(\frac{\ln x}{\ln 3}\right)}{\ln(x)}\right]$

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\left[\dfrac{\ln(\ln x) - \ln(\ln(3))}{\ln x}\right]$

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\left[\dfrac{\ln(\ln x)}{\ln x}\right] - \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\dfrac{\ln(\ln 3)}{\ln x}$

$\left[\dfrac{\ln(\ln x)}{\ln x}\right] \to \dfrac{\infty}{\infty}$; $\dfrac{\ln(\ln 3)}{\ln x} \to 0$

(LHopital's)

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2} \dfrac{\frac{1}{x}}{\frac{\ln x}{1}}$

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2} \dfrac{1}{\ln x}$

$= 0$

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or

$\lim_{x\to\infty} \dfrac{\log_2(\log_3 x)}{\log_5 x} = \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\left[\dfrac{\ln\left(\frac{\ln x}{\ln 3}\right)}{\ln x}\right]$ ($\ln\left(\frac{\ln x}{\ln 3}\right) \to \infty$)

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2}\left(\dfrac{(1/x)/\ln 3}{\ln x/\ln 3}\right)$ ($\dfrac{(1/x)/\ln 3}{\ln x/\ln 3} \to \infty$)

$= \lim_{x\to\infty} \dfrac{\ln 5}{\ln 2} \dfrac{1}{\ln x} = 0$

changing base: 10 pts (3 per change + 1 for not losing any constants)

Correctly applying LHopitals: 10 pts (5 per derivative)

Finding correct answer: 5 pts.