Math141 Exam 3 Problem 4

October 2014

Solution 1.

$$\lim_{n\to\infty}(\sin(1/n))^{1/n} = e^{\lim_{n\to\infty}(1/n)\ln(\sin(1/n))}$$

$$\lim_{n\to\infty}(1/n)\ln(\sin(1/n)) = \lim_{n\to\infty}\frac{\ln(\sin(1/n))}{n} \to \frac{-\infty}{\infty}$$

Use L'Hopital's rule.

$$\lim_{n\to\infty}\frac{\ln(\sin(1/n))}{n} = \lim_{x\to\infty}\frac{\ln(\sin(1/x))}{x}$$

$$\lim_{x\to\infty}\frac{\ln(\sin(1/x))}{x} = \lim_{x\to\infty}\frac{\frac{-\cos(1/x)}{x^2}}{\sin(1/x)} \to \frac{0}{0}$$

Use L'Hopital's rule again.

$$\lim_{x\to\infty}\frac{-\sin(1/x)(-1/x^2)(-1/x^2)+\cos(1/x)(2/x^3)}{\cos(1/x)(-1/x^2)} = 0$$

$$\lim_{n\to\infty}(\sin(1/n))^{1/n} = e^0$$

Solution 2.

$$\lim_{n\to\infty}(\sin(1/n))^{1/n} = \lim_{n\to\infty}(1/n)^{1/n}$$

$$\lim_{n\to\infty}(1/n)^{1/n} = e^{\lim_{n\to\infty}(1/n)\ln(1/n)}$$

Use L'Hopital

$$\lim_{x\to\infty}(1/x)\ln(1/x) = \lim_{x\to\infty}(1/x)\ln(1/x) = \lim_{x\to\infty}\frac{1}{x} = 0$$

$$\lim_{n\to\infty}(\sin(1/n))^{1/n} = e^0$$