John Benedetto's mathematical work

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Abstract John Joseph Benedetto (JB) has been at the University of Maryland, College Park since 1965. In this article I'll submit data that attests to JB's (a) large number of PhD students, (b) large number of papers[1], (c) outreach into the business sector, inviting cooperation between industry and his group of UMCP mathematicians that became the Norbert Wiener Center.

1 Brief biography

John ("Zip") Benedetto and Vienna DiTonno were both born and raised in Wakefield, a working class town just north of Boston, Mass. They were children of the depression and neither one got past 8th grade in school. But they met and they fell in love and, on June 17th, 1933, Vienna married Zip in Boston. They returned to Wakefield to settle down, and it was there, six years later, that their only child JB was born, on July 16th, 1939.

Zip[2] ran a pool hall in downtown Wakefield. While an excellent student at Malden Catholic High School, JB only got as far as trigonometry and solid geometry as they didn't teach calculus there in those days. After school, to his mom's dismay, JB would visit the pool hall almost daily to help his dad (and to play a little pool!). Another person who frequented Zip's pool hall was Robert McCloskey, a Harvard professor[3] and a collegiate billiards champion as an undergraduate. Seeing JB's academic talent, McCloskey told Zip to encourage JB to apply to Harvard. However,

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[1] As a linear regression computation shows, the number of PhD students (per year) he advises and the number of papers (published per year) are both increasing, on average. See below.

[2] Sadly, Zip passed away at age 44 in May of 1956, when JB was 16.

[3] According to archives of "The Crimson," McCloskey was appointed Chair of the Government Department at Harvard in 1958.

after graduation from high school in 1956 JB went to Boston College instead of Harvard. At Boston College, which he entered as a mathematics major[4], JB had an inspirational teacher for his first-year mathematics course, Fr. Stanley Bezuszka, S.J. (the chair of the mathematics department at the time). As a senior, encouraged by Fr. Bezuszka, and as a nod perhaps to McCloskey, JB applied to Harvard for graduate school, and nowhere else. Fortunately for mathematics, he was accepted and, after graduating from Boston College in 1960, began to take courses from Gleason (real analysis), Widder (Laplace transforms), Mackey, and Walsh (of Walsh functions fame), among others. His masters degree was awarded by Harvard in 1962.

In the fall of 1962, JB left Harvard for the University of Toronto, where he studied with Chandler Davis[5], who JB did not know of at Harvard. The reason for this move to Canada is not as simple as it sounds. It has really nothing to do with the fact that both Walsh and Davis had advisors in the Birkhoff family (both Harvard faculty members at the time). At Boston College and Harvard, JB was very interested in Thomistic philosophy (the philosophy of Thomas Aquinas) and he knew the Pontifical Institute of Medieval Studies at St. Michael's College was a subset of the University of Toronto. His plan was to get a PhD in mathematics in 1964 and a LMS from the Pontifical Institute[6] along the way. JB even knew what he wanted to work on for his PhD: the Laplace transform of distributions and topological vector spaces[7]. So, in the summer of 1962, JB is a man who knows what he wants. However, once JB arrived in Toronto that fall, life had other plans. First, JB was assigned as a TA to Chandler Davis. That's how they met and started working together. Second, he started taking classes at the Pontifical Institute, but after the first philosophy course dropped his plan to get an LMS. In fact, JB was Chandler Davis' first PhD student and they got along very well[8]. JB's PhD degree was awarded by the University of Toronto two years later, and a revised version of his thesis was published in [1966b].

In 1964, after graduating, JB took a tenure-track job at New York University[9]. While a graduate student, during the summers JB worked at RCA in Burlington MA. However, starting the summer of 1964 and part-time during the academic year, JB worked at IBM Cambridge, instead of RCA. On a whim, JB left NYU for a post-doc position at the Institute of Fluid Dynamics and Applied Mathematics at UMCP the following year. He became tenure track in the Department of Mathematics the following year. Except for visiting positions at MIT, the Mittag-Leffler Institute, and Scuola Normale Superiore, JB has been at UMCP since 1965. Once at UMCP, JB

[4] It was typical at the time for a BC student to declare their major as a freshman.

[5] From the it's-a-small-world department: Chandler Davis' PhD advisor was Garrett Birkhoff, son of George David Birkhoff, who was Joseph Walsh's PhD advisor.

[6] The LMS, a License in Mediaeval Studies or "Licentiate," is a kind of post-graduate degree awarded by The Pontifical Institute of Mediaeval Studies. There is no analogous degree offered in the U.S.

[7] Inspired by Widder's course on the Laplace transform and the reading course with Mackey on distributions and topological vector space that he took at Harvard.

[8] JB has written about his connection with Chandler Davis in [2014a].

[9] That year, the Courant Institute moved to its current location, in Weaver Hall.

Fig. 1 JB getting his PhD, with mom Vienna and grandpa Mr DiTonno in 1964.

continued to consult for industry but, of course, eventually this work came under the umbrella of the Norbert Wiener Center (more on that below).

As far as the arc of his career is concerned, JB's main mathematical inspirations are:

- Chandler Davis (PhD advisor),
- N. Wiener (who JB never met),
- A. Beurling (who JB never met),
- A. Gleason, one of his teachers at Harvard.

While JB has told me that many of the ideas he gets for papers are from thinking about mathematics while on a walk or traveling, I know there is another source: lots of hand computations. To illustrate this, I'll tell a story connected with my PhD thesis (in 1983, shortly after the publication of my favorite paper of his, [1980a]). As a graduate student, he assigned me a problem connected with his Mathematische Annalen 1980 paper. I don't remember the problem but I remember that after I solved it, I didn't want to take credit for it if he already solved it but just didn't, for whatever reason, add it to his paper. So one day we had a meeting in his office about this psychological problem I was having. He said to resolve the matter, I could read the notes he made while writing the paper. Apparently, for each paper JB writes, he keeps his notes (or at least, did at the time) in a notebook. So, JB pulls out this massive notebook (the kind with the extra large rings) full of hand-written computations. That JB keep such a massive set of detailed notes for each paper was amazing to me at the time, and still is.

Fig. 2 Chandler Davis and JB in 1964.

In his career, JB has been a Senior Fulbright-Hays Scholar, a SPIE Wavelet Pioneer, a Fellow of the American Mathematical Society, and a SIAM Fellow. His paper [1989b] won MITRE's Best Paper Award, and he was named Distinguished Scholar-Teacher by the University of Maryland in 1999.

Currently, JB is the Director of the Norbert Wiener Center for Harmonic Analysis and Applications (NWC), which he founded in 2004. It serves as an interface between funding agencies and industry with problems that can be solved using harmonic analysis by mathematicians at the NWC[10]. In its 15 years of existence, the NWC has brought in millions of dollars in grants and has worked with over 15 industrial partners[11]. Besides dollar grants, many of these industrial partners have also supported numerous student internships. Hundreds have been spoken or attended the annual NWC conference, the February Fourier Talks, or FFT. The NWC is also connected with the *Journal of Fourier Analysis and its Applications*[12] and the *Applied and Numerical Harmonic Analysis* book series[13].

As of summer 2019, JB has directed 58 Ph.D students (with several more in the pipeline). As of this writing, JB is in the top 100 of all PhD advisors world-wide[14]. JB does not co-author published PhD theses of his PhD students. Nonetheless, he has over 200 publications, of this writing, and over 80 co-authors (many of which are his former PhD students, if they do research with JB going beyond their thesis). What's

[10] Currently, JB, Radu Balan, Wojciech Czaja, and Kasso Okoudjou.

[11] For example, NIH, AFOSR, Siemens, MITRE, DARPA, ONR, NSF, and many more.

[12] For which JB is the Founding Editor-in-Chief.

[13] For which JB is the Founding Series Editor.

[14] According to the database "Mathematics Genealogy Project."

even more impressive is that none of JB's academic publications were co-authored until 1983. At the time of this writing, JB's most frequent co-author (by far) is his UMCP colleague, Wojciech Czaja.

2 Papers

The majority of mathematical papers by JB deal with the representation of an "arbitrary" function[15] (typically on $\mathbb{R}$ or $\mathbb{R}^n$ and subject to some conditions), in one way or another (by a Fourier series, wavelet expansion, integral transform, and so on). The functions JB considers can be pretty general, but the point is that JB represents them for us in a nice way and then uses such a representation to derive something useful. In many of his papers, JB takes such a representation and either (a) analyzes it to obtain estimates of a related quantity, or (b) applies it to solve an engineering problem, or (c) uses it to investigate a question in another field such as graph theory or analytic number theory.

Firstly, the list below includes some repetition (which I've tried to indicate). For example, some "technical reports" were revised and then submitted to a journal for publication. Secondly, some technical reports were not even submitted (for example, they might have a more expository flavor). Finally, we note that some papers have very similar, or even identical, titles but are essentially unrelated (unless indicated).

JB's papers

[1965a] *Representation theorem for the Fourier transform of integrable functions*, Bull. Soc. Roy. Sci. de Liege 9-10 (1965) 601-604.
[1965b] *Onto Criterion for adjoint maps*, Bull. Soc. Roy. Sci. de Liege 9-10 (1965) 605-609.
[1966a] **Generalized Functions**, Institute for Fluid Dynamics and Applied Math., BN-431 (1966) 1-380.
[1966b] *The Laplace transform of generalized functions*, Canad. J. Math. 18 (1966), 357-374.
[1967a] *Tauberian translation algebras*, Ann. di Mat. 74 (1967) 255-282.
[1967b] *Analytic representation of generalized functions,* Math. Zeit. 97 (1967), 303-319.
[1968a] **Pseudo-measures and Harmonic Synthesis**, University of Maryland, Department of Mathematics Lecture Notes 5 (1968) 1-316.
[1970a] *A strong form of spectral resolution*, Ann. di Mat. 86 (1970), 313-324.
[1970b] *Sets without true distributions*, Bull. Soc. Roy. Sci. de Liege 7-8 (1970), 434-437.
[1970c] *Support preserving measure algebras and spectral synthesis,* Math. Zeit. 118 (1970), 271-280.
[1971a] **Harmonic Analysis on Totally Disconnected Sets**, Lecture Notes in Mathematics, 202, Springer-Verlag, 1971.
[1971b] *Dirichlet series, spectral synthesis, and algebraic number fields*, University of Maryland, Department of Mathematics, TR 71-41 (1971), 1-23.

[15] Of course, the question "what's a function" immediately arises. Here we include both "generalized functions" (e.g., a distribution in the sense of Schwartz) and Radon measures as functions.

[1971c] *Dirichlet series, spectral synthesis, and algebraic number fields, Part I*, University of Maryland, Department of Mathematics, TR 71-41 (1971), 1-23.
[1971d] *Trigonometric sums associated with pseudo-measures*, Ann. Scuola Norm. Sup., Pisa 25 (1971), 229-248.
[1971e] *Sui problemi di sintesi spettrale*, Rend. Sem. Mat., Milano 4l (1971), 55-61.
[1971f] *Il Problema degli insiemi Helson-S*, Rend. Sem. Mat., Milano 41 (1971), 63-68.
[1971g] *(LF) spaces and distributions on compact groups and spectral synthesis on* $\mathbb{R}/2\pi\mathbb{Z}$, Math. Ann. 194 (1971), 52-67.
[1972a] *Measure zero: Two case studies*, TR, 1972, 26 pages.
[1972b] *Ensembles de Helson et synthese spectrale*, CRAS, Paris 274 (1972), 169-170.
[1972c] *Construction de fonctionnelles multiplicatives discontinues sur des algebres metriques*, CRAS, Paris 274 (1972), 254-256.
[1972d] *A support preserving Hahn-Banach property to determine Helson-S Sets*, Inventiones Math. 16 (1972), 214-228.
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[1974a] *Pseudo-measure energy and spectral synthesis*, Can. J. Math. 26 (1974), 985-1001.
[1974b] *Tauberian theorems, WienerâĂŹs spectrum, and spectral synthesis*, Rend. Sem. Mat., Milano 44 (1974) 63-73.
[1975a] **Spectral Synthesis**, Pure and applied mathematics series, vol. 66, Academic Press, N.Y., 1975.
[1975b] *Zeta functions for idelic pseudo-measures*, University of Maryland, Department of Mathematics, TR 74-55 (1975), 1-46.
Appeared as [1979a].
[1975c] *The Wiener spectrum in spectral synthesis*, Studies in Applied Math. (MIT) 54 (1975) 91-115.
[1977a] *Analytic properties of idelic pseudo-measures*, University of Maryland, Department of Mathematics, TR 77-62 (1977), 1-33.
[1977b] *Idelic pseudo-measures and Dirichlet series*, Symposia Mathematica, Academic Press, 1976 Conference on Harmonic Analysis, Rome 22 (1977), 205-222.
[1979a] *Zeta Functions for idelic pseudo-measures*, Ann. Scuola Norm. Sup., Pisa 6 (1979) 367-377.
[1980a] *Fourier analysis of Riemann distributions and explicit formulas*, Math. Ann. 252 (1980), 141-164.
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[1981b] *Spectral deconvolution*, University of Maryland, Department of Mathematics, TR 81-63 (1981), 1-25.
[1981c] *The theory of constructive signal analysis*, Studies in Applied Math. (MIT) 65 (1981), 37-80.
[1981d] *Wiener's Tauberian theorem and the uncertainty principle*, Proc. of Modern Harmonic Analysis Conference, Torino-Milano (1982), (1983) 863-887.
[1982a] *A closure problem for signals in semigroup invariant systems*, SIAM J. Math. Analysis 13 (1982) 180-207.
[1983a] *Estimation problems and stochastic image analysis* (with S. Belbas), University of Maryland, Department of Mathematics, TR89-67 (1983), 1-15.
[1983b] *Harmonic analysis and spectral estimation*, J. Math. Analysis and Applications 91 (1983) 444-509.
[1983c] *Weighted Hardy spaces and the Laplace transform* (with H. Heinig), Lecture Notes in Mathematics, 992, Springer Verlag (Cortona Conference, 1982), (1983), 240-277.
[1983d] *WienerâĂŹs Tauberian theorem and the uncertainty principle*, Proc. of Modern Harmonic Analysis Conference, Torino-Milano (1982), (1983), 863-887.
[1984a] *A local uncertainty principle*, SIAM, J. Math. Analysis 15 (1984) 988-995.
[1985a] *An inequality associated with the uncertainty principle*, Rend. Circ. Mat. di Palermo 34 (1985) 407-421.

[1985b] *Some mathematical methods for spectrum estimation*, in **Fourier Techniques and Applications**, J.F. Price, editor, Plenum Publishing (1985) 73-100.
[1985c] *Fourier uniqueness criteria and spectrum estimation theorems*, in **Fourier Techniques and Applications**, J.F. Price, editor, Plenum Publishing (1985) 149-172.
[1986a] *Inequalities for spectrum estimation*, Linear Algebra and Applications 84 (1986), 377-383.
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[1986c] *Fourier inequalities with A_p weights* (with H. Heinig and R. Johnson), General Inequalities 5, Oberwolfach, (1986), ISNM 80 (1987), 217-232.
[1987a] *A quantitative maximum entropy theorem for the real line*, Integral Equations and Operator Theory 10 (1987) 761-779.
[1989a] *Gabor representations and wavelets*, AMS Contemporary Mathematics, 91 (1989) 9-27.
[1989b] *The Wiener-Plancherel formula in Euclidean space*, (with G. Benke and W. Evans), Advances in Applied Math., 10 (1989) 457-487.
Note: This won the The Best Paper Award from the MITRE Corporation.
[1990a] *Heisenberg wavelets and the uncertainty principle*, Prometheus Inc., TR (1990), 1-3.
[1990b] *Wavelet auditory models and irregular sampling*, Prometheus Inc., TR (1990), 1-6.
[1990c] *Uncertainty principle inequalities and spectrum estimation*, NATO-ASI, in **Fourier Analysis and its Applications 1989**, J. Byrnes, editor, Kluwer Publishers, The Netherlands, Series C, 315 (1990) 143-182.
[1990d] *Irregular sampling and the theory of frames, I* (with W. Heller), Mat. Note, 10, Suppl.n.1 (1990) 103-125.
(Note: There is no part II. It is mostly independent of [1992a] and [1992f].)
[1991a] *Support dependent Fourier transform norm inequalities*, (with C. Karanikas), Rend. Sem. Mat., Roma, 11 (1991) 157-174.
[1991b] *The spherical Wiener-Plancherel formula and spectral estimation*, SIAM Math. Analysis, 22 (1991) 1110-1130.
[1991c] *Fourier transform inequalities with measure weights II* (with H. Heinig), Second International Conference on Function Spaces 1989, Poznan, Poland, Teubner Texte zur Mathematik series, 120 (1991) 140-151.
[1991d] *A multidimensional Wiener-Wintner theorem and spectrum estimation*, Trans. AMS, 327 (1991) 833-852.
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[1992b] *Stationary frames and spectral estimation*, NATO-ASI, in **Probabilistic and Stochastic Methods in Analysis, with Applications 1991**, J. Byrnes, editor, Kluwer Publishers, The Netherlands, Series C, (1992) 117-161.
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[1992e] *Fourier transform inequalities with measure weights* (with H. Heinig), Advances in Math., 96 (1992) 194-225.
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[1993b] *On frames and filter banks* (with S. Li), Conference on Information Sciences and Systems, at The Johns Hopkins University, Technical Co-Sponsorship with IEEE, 1993, invited.
[1993c] *A wavelet auditory model and data compression* (with A. Teolis), Applied and Computational Harmonic Analysis, 1(1993) 3-28.
[1993d] *Local frames* (with A. Teolis), SPIE, Mathematical Imaging: Wavelet Applications in Signal and Image Processing, 2034 (1993) 310-321.

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[1994a] *Noise reduction filters in mathematical models of biological systems*, The MITRE Corp., TR (1994), 1-6.

[1994b] *Noise reduction using frames, irregular sampling, and wavelets*, ORD Signal Processing, Fort Meade, MD, 1994.

[1994c] *Frame decompositions, sampling, and uncertainty principle inequalities*, in **Wavelets: Mathematics and Applications**, J. Benedetto and M. Frazier, editors, CRC Press, Boca Raton, FL, (1994) 247-304.

[1994d] *Gabor frames for L^2 and related spaces* (with D. Walnut), in **Wavelets: Mathematics and Applications**, J. Benedetto and M. Frazier, editors, CRC Press, Boca Raton, FL (1994) 97-162.

[1994e] *Noise suppression using a wavelet model* (with A. Teolis), IEEE - ICASSP (1994).

[1994f] *Subband coding for sigmoidal nonlinear operations* (with S. Saliani), SPIE, Wavelet Applications, 2242 (1994) 19-27.

[1994g] *Subband coding and noise reduction in multiresolution analysis frames* (with S. Li), SPIE, Wavelet Applications in Signal and Image Processing II, 2303 (1994) 154-165.

[1994h] *The definition of the Fourier transform for weighted inequalities* (with J. Lakey), J. of Functional Analysis, 120 (1994) 403-439.

[1994i] *Analysis and feature extraction of epileptiform EEG waveforms* (with D. Colella, G. Jacyna, et al.), Fifth International Cleveland Clinic - Bethel Epilepsy Symposium, 1994, Poster.

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[1995c] *Wavelet-based analysis of EEG signals for detection and localization of epileptic seizures* (with G. Benke, M. Bozek-Kuzmicki, D. Colella, G. Jacyna), SPIE, Wavelet Applications for Dual Use, 2491 (1995) 760-769.

[1995d] *Differentiation and the Balian-Low theorem* (with C. Heil and D. Walnut), J. of Fourier Analysis and Applications, 1 (1995) 355-402.

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[1995f] *Local frames and noise reduction* (with A. Teolis), Signal Processing, 45 (1995) 369-387.

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[1997a] *Generalized harmonic analysis and Gabor and wavelet systems*, AMS Proceedings of Symposia in Applied Mathematics, volume 52, Wiener Centenary Volume, 1997, pages 85-113, invited.

[1997b] *Sampling operators and the Poisson summation formula*, (with G. Zimmermann), Journal of Fourier Analysis and Applications, 3 (1997) 505-523.

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[1998c] *Self-similar pyramidal structures and signal reconstruction* (with M. Leon and S. Saliani), SPIE, Wavelet Applications V, 3391 (1998) 304-314, invited.
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[1998e] *Frames, sampling, and seizure prediction*, in **Advances in Wavelets**, K.-S. Lau, editor, Springer-Verlag, New York, 1998, Chapter 1, pages 1-25, invited.
[1998f] *Wavelet periodicity detection algorithms* (with G. Pfander), SPIE, Wavelet Applications in Signal and Image Processing VI, 3458 (1998), 48-55, invited.
[1999a] *A multidimensional irregular sampling algorithm and applications* (with H.-C. Wu), IEEE - ICASSP, Phoenix, Special Session on Recent Advances in Sampling Theory and Applications, 4 (1999), 4 pages, invited.
[1999b] *The construction of multiple dyadic minimally supported frequency wavelets on* $\mathbb{R}^d$ (with M. Leon), AMS Contemporary Math. Series, 247 (1999) 43-74.
[1999c] *A Beurling covering theorem and multidimensional irregular sampling* (with H.-C. Wu), in **Sampling Theory and Applications**, Loen, Norway, sponsored/published by Norwegian University of Science and Technology, (1999) 142 – 148, invited.
[2000a] *Sampling theory and wavelets*, NATO-ASI, in **Signal Processing for Multimedia 1998**, J. Byrnes, editor, Kluwer Publishers, The Netherlands, 2000, invited.
[2000b] *Ten books on wavelets*, SIAM Review, 42 (2000), 127-138. (Although not a research paper, I was asked to write an extensive review of recent books on wavelet theory, and involves input from several of my graduate students.)
[2000c] *Non-uniform sampling theory and spiral MRI reconstruction* (with H.-C. Wu), SPIE, Wavelet Applications in Signal and Image Processing VIII, 4119 (2000), invited.
[2001a] *The classical sampling theorem, and non-uniform sampling and frames* (with P.S.J.G. Ferreira), Chapter 1 of **Modern Sampling Theory: Mathematics and Applications**, J.J. Benedetto and P.S.J.G. Ferreira, editors, Birkhauser Boston, 2001.
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[2001c] *Wavelet frames: multiresolution analysis and extension principles* (with O. Treiber), Chapter 1 of **Wavelet Transforms and Time-Frequency Signal Analysis**, L. Debnath, editor, Birkhauser, Boston, 2001, pages 3 - 36, invited.
[2001d] *The construction of single wavelets in* d*-dimensions* (with M. Leon), J. Geometric Analysis, 11 (2001) 1-15.
[2002a] *MRI signal reconstruction by Fourier frames on interleaving spirals* (with A. Powell and H.-C. Wu), IEEE - ISBI 2002, 4 pages, invited.
[2002b] *A fractal set constructed from a class of wavelet sets* (with S. Sumetkijakan), AMS Contemporary Math. Series, 313 (2002) 19-35.
[2002c] *Periodic wavelet transforms and periodicity detection* (with G. Pfander), SIAM J. Applied Math., 62 (2002) 1329-1368.
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[2003b] *The Balian-Low theorem and regularity of Gabor systems* (with W. Czaja, P. GadziÅĎski, and A. Powell), J. Geometric Analysis, 13 (2003) 239-254.
[2003c] *Weighted Fourier inequalities: new proof and generalization* (with H. Heinig), J. Fourier Analysis and Applications, 9 (2003) 1-37.
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[2003e] *Local sampling for regular wavelet and Gabor expansions* (with N. Atreas and C. Karanikas), Sampling Theory in Signal and Image processing, 2 (2003) 1-24
[2003f] *A Wiener-Wintner theorem for* $1/f$ *power spectra* (with R. Kerby and S. Scott), J. Math. Analysis and Applications, 279 (2003) 740-755.

[2004a] Software package for CAZAC code generators and doppler shift analysis (with J. Donatelli and J. Ryan), 2004, see `http://www.math.umd.edu/~jjb/cazac`.

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[2004c] *Constructive approximation in waveform design* (invited) in **Advances in Constructive Aproximation Theory**, M. Neamtu and E. B. Saff,editors, Nashboro Press, (2004) 89-108.

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[2004e] *Sigma-Delta quantization and finite frames* (with A. Powell and ÃŰ. Yilmaz), ICASSP, Montreal, 2004, invited.

[2005a] *Multiscale Riesz products and their support properties* (with E. Bernstein and I. Konstantinidis), Acta Applicandae Math, 88(2) (2005) 201-227.

[2005b] *Analog to digital conversion for finite frames* (with A. Powell and ÃŰ. Yilmaz), SPIE, Wavelet Applications in Signal and Image Processing (2005), invited.

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[2006c] *Sigma-Delta quantization and finite frames* (with A. Powell and ÃŰ. Yilmaz), IEEE Trans. Information Theory, 52(5) (2006) 1990-2005.

[2006d] *Introduction* for **Fundamental Papers in Wavelet Theory**, edited by C. Heil and D. F. Walnut, Princeton University Press, 2006. (Although not a research paper, this is an extensive, 20 page, introduction for an important volume.)

[2006e] *An endpoint $(1, \infty)$ Balian-Low theorem* (with W. Czaja, A. Powell, and J. Sterbenz), Math. Research Letters, 13 (2006), 467-474.

[2006f] *An optimal example for the Balian-Low uncertainty principle*, (with W. Czaja and A. Powell), SIAM Journal of Mathematical Analysis, 38 (2006) 333-345.

[2006g] *Zero autocorrelation waveforms: a Doppler statistic and multifunction problems* (with J. Donatelli, I. Konstantinidis, and C. Shaw), ICASSP, Toulouse, 2006, invited.

[2006h] *A Doppler statistic for zero autocorrelation waveforms* (with J. Donatelli, I. Konstantinidis, and C. Shaw), Conference on Information Sciences and Systems, at Princeton University, Technical Co-Sponsorship with IEEE, (2006), 1403 – 1407, invited.

[2006i] *Frame expansions for Gabor multipliers* (with G. Pfander), Applied and Computational Harmonic Analysis, 20 (2006) 26-40.

[2006j] *Second order Sigma-Delta quantization of finite frame expansions* (with A. Powell and ÃŰ. Yilmaz), Applied and Computational Harmonic Analysis, 20 (2006) 128-148.

[2007a] *Ambiguity and sidelobe behavior of CAZAC waveforms* (with A. Kebo, I. Konstantinidis, M. Dellomo, J. Sieracki), IEEE Radar Conference, Boston,(2007).

[2007b] *The construction of d-dimensional MRA frames* (with J. Romero), J. Applied Functional Analysis, 2 (2007) 403 – 426.

[2007c] *Ambiguity function and frame theoretic properties of periodic zero autocorrelation functions* (with J. Donatelli), IEEE J. of Selected Topics in Signal Processing, 1 (2007) 6 – 20.

[2007d] *Target tracking using particle filtering and CAZAC sequences* (with I Kyriakides, I. Konstantinidis, D. Morrell, A. Papandreou-Suppappola), IEEE International Waveform Diversity and Design, Pisa, (2007), invited.

[2007e] *Concatenating codes for improved ambiguity behavior* (with A. Bourouihiya, I. Konstantinidis, and K. Okoudjou), Adaptive Waveform Technology for Futuristic Communications, Radar, and Navigation Systems, International Conference on Electromagnetics in Advanced Applications (ICEAA), Torino, 2007, invited.

[2008a] *The role of frame force in quantum detection* (with A. Kebo), J. Fourier Analysis and Applications, 14 (2008) 443 – 474.

[2008b] *PCM-Σ–Δ comparison and sparse representation quantization* (with O. Oktay), Conference of Information Science and Systems, Princeton, 2008, 6 pages, Invited.

[2008c] *Multiple target tracking using particle filtering and multicarrier phase-coded CAZAC sequences* (with I. Kyriakides, A. Papandreou-Suppappola, D. Morrell, and I. Konstantinidis), Sensor, Signal and Information Processing Workshop (SenSIP) 2008, Sedona AZ.

[2008d] *Human electrocortigraphic signature determination by eGAD sparse approximation* (with N. Crone and J. Sieracki), Sensor, Signal and Information Processing Workshop (SenSIP) 2008, Sedona AZ.

[2008e] *Complex Sigma-Delta quantization algorithms for finite frames* (with O. Oktay and A. Tangboondouangjit), AMS Contemporary Mathematics, 464 (2008)27-49.

[2008f] *Frames and a vector-valued ambiguity function* (with J. Donatelli), IEEE - Asilomar, 2008, invited.

[2009a] *Phase coded waveforms and their design - the role of the ambiguity function* (with I. Konstantinidis and M. Rangaswamy), IEEE Signal Processing Magazine (invited), 26 (2009) 22-31.

[2009b] *Hadamard matrices and infinite unimodular sequences with 0-autocorrelation* (with S. Datta), IEEE International Waveform Diversity and Design, Orlando, (2009), invited, but withdrawn since we couldnâĂŹt attend the conference.

[2009c] *Frame based kernel methods for automatic classification in hyperspectral data* (with W. Czaja, C. Flake, and M. Hirn), Proceedings IEEE-IGARSS 2009, invited.

[2009d] *Smooth functions associated with wavelet sets on* $\mathbb{R}^d$, $d \geq 1$, *and frame bound gaps* (with E. King), Acta Applicandae Math, 107 (2009) 121 – 142.

[2009e] *Geometric properties of Shapiro-Rudin polynomials* (with J. Sugar-Moore), Involve - a journal of mathematics, 2(4)(2009) 449 – 468.

[2010a] *Besov spaces for the SchrÃűdinger operator with barrier potential* (with S. Zheng), Complex Analysis and Operator Theory, 4(4)(2010), 777 – 811.

[2010b] *Pointwise comparison of PCM and Sigma-Delta quantization* (with O. Oktay), Constructive Approximation, 32(1)(2010), 131 – 158.

[2010c] *Construction of infinite unimodular sequences with zero autocorrelation* (with S. Datta), Advances in Computational Mathematics, 32 (2010), 191 – 207.

[2010d] *Wavelet packets for multi- and hyper-spectral imagery* (with W. Czaja, M. Ehler, C. Flake, and M. Hirn), Wavelet Applications in Industrial Processing VII, Proc. SPIE, 7535 (2010) 8 – 11, San Jose.

[2010e] *Frame potential classification algorithm for retinal data* (with W. Czaja and M. Ehler), 26th Southern Biomedical Engineering Conference, College Park, MD 2010.

[2010f] *Maximally separated frames for automatic classification in hyperspectral data* (with W. Czaja, M. Ehler, and N. Strawn), IGARSS 2010, Honolulu, preprint.

[2011a] *The construction of wavelet sets* (with R. Benedetto) in **Wavelets and Multiscale Analysis**, J. Cohen and A. I. Zayed, editors, BirkhÃđuser-Springer 2011, Chapter 2, pages 17–56.

[2011b] *Discrete autocorrelation-based multiplicative MRAs and sampling on* $\mathbb{R}$ (with. S. Datta), Sampling Theory in Signal and Image Processing 10 (2011) 111 – 133.

[2011c] *Intrinsic wavelet and frame applications* (with T. Andrews), invited paper, SPIE 2011, Orlando.

[2012a] *Image representation and compression via sparse solutions of systems of linear equations* (with A. Nava-Tudela), Technical Report, 2012. (Revised and published in [2014f].)

[2012b] *Optimal ambiguity functions and WeilâĂŹs exponential sums bound* (with R. L. Benedetto and J. Woodworth), J. Fourier Analysis and Applications, 18 (2012) 471–487.

[2012c] *Constructions and a generalization of perfect autocorrelation sequences on* $\mathbb{Z}$ (with Somantika Datta), invited chapter in volume dedicated to Gil Walter, X. Shen and A. Zayed, eds., Chapter 8, Springer (2012) 183 – 207.

[2012d] *Integration of heterogeneous data for classification in hyperspectral satellite imagery* (with W. Czaja, J. Dobrosotskaya, T. Doster, K. Duke, and D. Gillis), SPIE 2012, Baltimore.

[2012e] *Semi-supervised learning of heterogeneous data in remote sensing imagery* (with W. Czaja, J. Dobrosotskaya, T. Doster, K. Duke, and D. Gillis), invited paper SPIE 2012, Baltimore.
[2013a] *Balayage and short time Fourier transform frames* (with E. Au-Yeung), SampTA 2013 at Bremen, invited.
[2013b] *Wavelet packets for time-frequency analysis of multi-spectral images* (with W. Czaja and M. Ehler), International J. of Geomathematics, 4 (2013) 137–154.
[2014a] *Chandler Davis as mentor*, Mathematical Intelligencer, 36 (2014) 20–21.
[2014b] *Wavelet packets and nonlinear manifold learning for analysis of hyperspectral data* (with W. Czaja, T. Doster, and C. Schwartz), SPIE 2014, Baltimore.
[2014c] *Operator-based integration of information in multimodal radiological search mission with applications to anomaly detection* (with A. Cloninger, W. Czaja, T. Doster, B. Manning, T. McCullough, K. Kochersbeger, and M. McLean), Proc. SPIE 9073, Chemical, Biological, Radiological, Nuclear, and Explosives (CBRNE) Sensing XV, Baltimore, 2014.
[2014d] *Nonlinear dimensionality reduction via the ENH–LTSA method for hyperspectral image classification* (with W. Czaja, A. Halevy, W. Li, C. Liu, B. Shi, W. Sun, R. Wang), IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing (IEEE JSTARS), 7 (2014) 375–388.
[2014e] *UL-Isomap based dimensionality reduction for hyperspectral imagery classification* (with W. Czaja, A. Halevy, W. Li, C. Liu, B. Shi, W. Sun, H. Wu), Journal of Photogrammetry and Remote Sensing (ISPRS), 89 (2014) 25–36.
[2014f] *Sampling in image representation and compression* (with A. Nava-Tudela), Sampling Theory in Honor of Paul Bützer on his 85th Birthday, A. Zayed, editor, invited Chapter, Springer-Birkhauser, 2014, Chapter 7, pages 149–188.
[2015a] *Fourier operators in applied harmonic analysis* (with M. BeguÃľ), in **Sampling Theory – a Renaissance**, G. Pfander, editor, invited Chapter, Springer-Birkhauser, Chapter 5, 2015, 185–216.
[2015b] *The HRT conjecture for functions with certain behavior at infinity* (with A. Bourouihiya), J. Geometric Analysis, accepted on-line 2014, 25 (2015) 226–254.
[2015c] *Dimension reduction and remote sensing using modern harmonic analysis* (with W. Czaja), in **Handbook of Geomathematics**, edited by W. Freeden, Z. Nashed, and T. Sonar), invited Chapter, Springer, New York, 2015, pages 2609 – 2632.
[2015d] *Generalized Fourier frames in terms of balayage* (with E. Au-Yeung), J. Fourier Analysis and Applications, 21 (2015) 472–508.
[2015e] *Graph theoretic uncertainty principles* (with P. Koprowski), SampTA 2015 Washington D.C., invited, 2015.
[2015f] *Balayage and pseudo-differential equation frame inequalities* (with E. Au-Yeung), SampTA 2015 Washington D.C., 2015.
[2016a] *Spatial-spectral operator theoretic methods for hyperspectral image clasification* (with W. Czaja, J. Dobrosotskaya, T. Doster, and K. Duke), International Journal on Geomathematics (GEM) 7 (2016) 275–297.
[2016b] *Preface for* **Finite Frames**, K. Okoudjou, editor, AMS Symposia in Mathematics (based on AMS Workshop at JMM 2015), 2016.
[2017a] *Uncertainty principles and weighted norm inequalities* (with M. Dellatorre), invited chapter in **Functional Analysis, Harmonic Analysis, and Image Processing: a Collection of Papers in Honor of Björn Jawerth**, Michael Cwikel and Mario Milman, editors, AMS Contemporary Mathematics 693 (2017), 55–78.
[2017b] *A frame reconstruction algorithm with applications to MRI* (with A. Nava-Tudela, A. Powell, and Y. Wang), invited chapter, Chapter 9 in **Frames and Other Bases in Abstract and Function Spaces: Novel Methods in Harmonic Analysis, Volume 1**, I. Pesenson, et al. editors, Springer-BirkhÃďuser, New York, 2017.
[2017c] *Super-resolution by means of Beurling minimal extrapolation* (with W. Li), Applied and Computational Harmonic Analysis, Online publication 2018,
arXiv: 1601.05761v3

[2017d] *Frame multiplication theory and a vector-valued DFT and ambiguity function* (with T. Andrews and J. Donatelli), J. Fourier Analysis and Appl., Online publication 2018. arXiv: 1706.05579v1

This is an average of about 3.48 papers per year. The list of pairs (year, number of papers published that year) between 1965 and 2017 has best linear fit[16] $y = ax + b$, where $a = 0.0708...$, $b = -137.6162....$ In rough terms, the number of papers JB publishes per year increases by about 7% per year. The graph is in Figure 3.

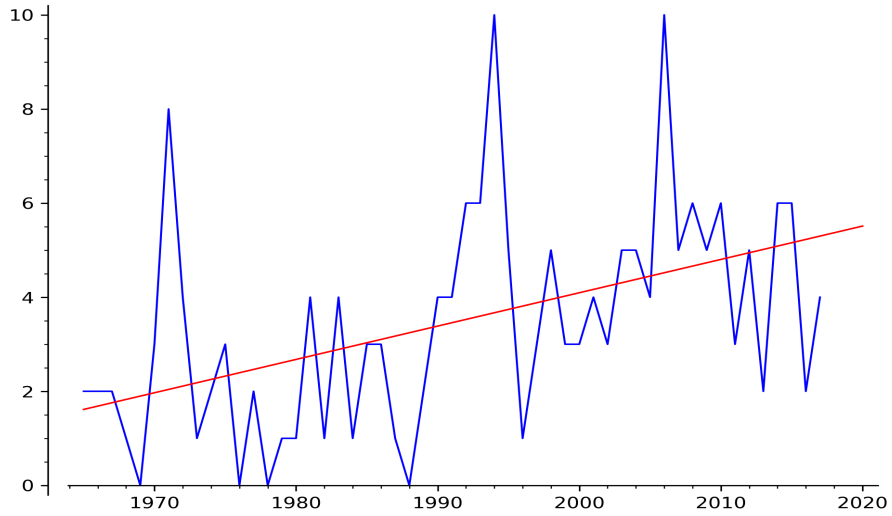

Fig. 3 Linear regression on the number of JB's papers published per year.

3 PhD theses

Here is a list of the 58 (and counting) PhD students that JB has advised, as of this writing.

[16] Thanks to the SageMath command `find_fit`.

JB's PhD students

[1971] George Benke, *Sidon sets and the growth of L^p norms*
[1977a] Wan-chen Hsieh, *Topologies for spectral synthesis of the space of bounded functions*
[1977b] Fulvio Ricci, *Support preserving multiplication of pseudo-measures*
[1980] Ward Evans[17], *Beurling's spectral analysis and continuous pseudo-measures*
[1983] W. David Joyner, *The harmonic analysis of Dirichlet series and the Riemann zeta function*
NSF Post Doc IAS 1984
[1987] Jean-Pierre Gabardo, *Spectral gaps and uniqueness problems in Fourier analysis*
Sloan Dissertation Fellowship 1986
[1989] David Walnut, *Weyl-Heisenberg wavelet expansions: existence and stability in weighted spaces*
Sloan Dissertation Fellowship 1988
[1990a] Christopher Heil, *Wiener amalgam spaces in generalized harmonic analysis and wavelet theory*
NSF Post Doc MIT 1990
[1990b] Rodney Kerby, *Correlation function and the Wiener-Wintner theorem in higher dimensions*
[1990c] George Yang, *Applications of Wiener-Tauberian theorem to a filtering problem and convolution equations*
[1991a] William Heller, *Frames of exponentials and applications*
[1991b] Joseph Lakey, *Weighted norm inequalities for the Fourier transform*
[1992] Erica Bernstein, *Generalized Riesz products and pyramidal schemes*
[1993a] Shidong Li, *The theory of frames and filter design*
[1993b] Sandra Saliani, *Nonlinear wavelet packets*
[1993c] Anthony Teolis, *Discrete signal representation*
[1994] Georg Zimmermann, *Projective multiresolution analysis and generalized sampling*
[1998a] Melissa Harrison, *Frames and irregular sampling from a computational perspective*
[1998b] Hui-Chuan Wu, *Multidimensional irregular sampling in terms of frames*
[1999a] Manuel Leon, *Minimally supported frequency wavelets wavelets*
[1999b] Götz Pfander, *Periodic wavelet transforms and periodicity detection*
[1999c] Oliver Treiber, *Affine data representations and filter banks*
[2000] Sherry Scott, *Spectral analysis of fractal noise in terms of Wiener's Generalized Harmonic Analysis and wavelet theory*
[2001a] Matthew Fickus, *Finite normalized tight frames and spherical equidistribution*
[2001b] Ioannis Konstantinidis, *The characterization of multiscale generalized Riesz product measures*
[2002a] Anwar A. Saleh, *A finite dimensional model for the inverse frame operator*
[2002b] Jeffrey Sieracki, *Greedy adaptive discrimination: Signal component analysis by simultaneous matching pursuit with application to ECoG signature detection*
[2002c] Songkiat Sumetkijakan, *A fractal set constructed from a class of wavelet sets*
[2003a] Alexander M. Powell, *The uncertainty principle in harmonic analysis and BourgainâĂŹs theorem*
Dissertation Fellowship 2000
[2003b] Shijun Zheng, *Besov spaces for the Schrödinger operator with barrier potential*
Dissertation Fellowship 2000
[2004] Joseph Kolesar, Σ–Δ *modulation and correlation criteria for the construction of finite frames arising in communication theory*
[2005a] Andrew Kebo, *Quantum detection and finite frames*
[2005b] Juan Romero, *Generalized multiresolution analysis: construction and measure theoretic characterization*
[2006a] Abdelkrim Bourouihiya, *Beurling weighted spaces, product-convolution operators, and the tensor product of frames*

[17] Now named Celia Evans.

[2006b] Aram Tangboondouangjit, *Sigma-Delta quantization: number theoretic aspects of refining error estimates*
[2007a] Somantika Datta, *Wiener's Generalized Harmonic Analysis and wave-form design*
[2007b] Onur Oktay, *Frame quantization theory and equiangular tight frames*
[2008] David Widemann, *Dimensionality reduction for hyperspectral data* (Co-adviser, W. Czaja)
[2009a] Matthew Hirn, *Enumeration of harmonic frames and frame based dimension reduction* (Co-adviser, K. Okoudjou)
[2009b] Emily King, *Wavelet and frame theory: frame bound gaps, generalized shearlets, Grassmannian fusion frames, and p-adic wavelets* (Co-adviser, W. Czaja)
Wylie Dissertation Fellowship 2008
[2010] Christopher Flake, *The multiplicative Zak transform, dimension reduction, and wavelet analysis of LIDAR data* (Co-adviser, W. Czaja)
[2011a] Enrico Au-Yeung, *Balayage of Fourier transforms and the theory of frames*
[2011b] Avner Halevy, *Extensions of Laplacian eigenmaps for manifold learning* (Co-adviser W.Czaja)
[2011c] Nathaniel Strawn, *Geometric structures and optimization on finite frames* (Co-adviser, R. Balan)
[2012a] Kevin Duke, *A study of the relationship between spectrum and geometry through Fourier frames and Laplacian Eigen-maps*
[2012b] Alfredo Nava-Tudela, *Image representation and compression via sparse solutions of systems of linear equations*
[2013] Rongrong Wang, *Global geometric conditions on dictionaries for the convergence of ℓ^1 minimization problems* (Co-adviser W.Czaja)
[2014a] Travis Andrews, *Frame multiplication theory for vector-valued harmonic analysis*
[2014b] Alex Cloninger, *Exploiting data-dependent structure for improving sensor acquisition and integration* (Co-adviser W. Czaja)
Wylie Dissertation Fellowship 2013
NSF Postdoctoral Fellowship to Yale
[2014c] Tim Doster, *Harmonic analysis inspired data fusion with applications in remote sensing* (Co-adviser W.Czaja)
[2014d] Wei-Hsuan Yu, *Spherical two-distance sets and related topics in harmonic analysis* (Co-adviser A. Barg)
[2015a] Gokhan Civan, *Identification of operators on elementary locally compact abelian groups*
[2015b] Paul Koprowski, *Graph-theoretic uncertainty principles*
[2015c] James Murphy, *Anisotropic harmonic analysis and integration of remotely sensed data* (Co-advisor W. Czaja)
[2016] Matthew Begué, *Expedition in data and harmonic analysis on graphs* (Co-advisor K. Okoudjou)
[2018a] Weilin Li, *Topics on harmonic analysis, sparse representations, and data analysis* (Co-advisor W. Czaja) Wylie Dissertation Fellowshipc 2017
[2018b] Mark Magsino, *Constant amplitude zero-autocorrelation sequences and single pixel camera imaging*
[2018c] Franck Njeunje, *Computational methods in machine learning: transport model, Haar wavelet, DNA classification, and MRI* (Co-advisor W. Czaja)

This is an average of about 1.2 PhD students per year. The list of pairs (year, number of JB's PhD students graduating that year) between 1971 and 2017 has best linear fit[18] $y = ax + b$, where $a = 0.0451...$, $b = -88.8937....$ In rough terms, the number of PhD students JB graduates per year increases by about 4.5% per year. The graph is in Figure 4.

[18] Again, thanks to SageMath.

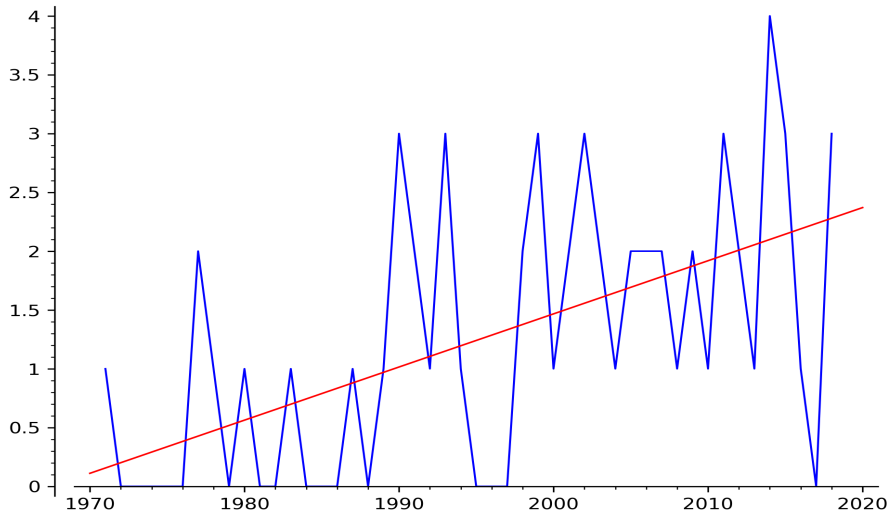

Fig. 4 Linear regression on the number of JB's PhD students graduating per year.

4 Coda

In summary, JB's piercing intellectual curiosity has led to over 200 hundred refereed publications and about 60 PhD students, so far. Which reminds me of the old joke, "Great mathematicians never die, they just tend to infinity."