MATH 734

MIDTERM EXAM

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1. Assume that A and B are acyclic, that $W = A \bigcup B = A^o \bigcup B^o$ and that $X = A \bigcap B$. Calculate the homology of W in relation to X.

2. a) Suppose $f : S^n \to S^n$ satisfies $f(-x) = f(x)$ for all x, where n is even. Show that $\deg f = 0$.

b)Find a counterexample for n odd.

c) Let $f, g : S^n \to S^n$ be such that $f(x) \neq g(x)$ for all $x \in S^n$. Show $\deg f = (-1)^{n+1} \deg g$.

3. Let C_* be a chain complex whose homology is of finite rank (i.e. all homology groups are of finite rank and almost all are of rank 0). Then we can define its Euler characteristic as $\chi(C_*) = \sum (-1)^n rk(H_n(C_*))$. Assume that

$$0 \to A_* \to B_* \to C_* \to 0$$

is an exact sequence of chain complexes. Assuming that all three have homologies of finite rank. Prove that $\chi(B_*) = \chi(A_*) + \chi(C_*)$.

4. Let $A, B \subseteq S^n$, $n \geq 2$, be closed subspaces homeomorphic to S^1.

Suppose $A \cap B$ consists of two distinct points.

a) If $n = 2$ show $S^n - (A \cup B)$ has 4 components and each component C is acyclic, i.e. $\tilde{H}_*(C) = 0$.
b) If $n \geq 3$, show $S^n - (A \cup B)$ has one component C and $H_*(C) \approx H_*(S^{n-2} \vee S^{n-2} \vee S^{n-2})$.

You may use the Generalized Jordan Curve Theorem.

5. Let $0 \leq n \leq m$ and let X be the CW-complex with one zero cell, one 1-cell and two 2-cells attached by maps $f, g : S^1 \to S^1$ of degree n and m respectively.

a) Write down the cellular chain complex of X including the boundary maps.
b) Compute the homology of X.
c) Show the fundamental group of X is a finite cyclic group and determine its order.
d) Compute the homology groups of $\widetilde{X}$, the universal cover of X.