Corrections to “Intro. to Homological Algebra” by C. Weibel

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p.2 line -12: d_{n-1} should be d_n

p.4 lines 5,6: $V - E - 1$ should be $E - V + 1$ (twice)

p.4 lines 7,8: all 5 occurrences of v_0 should be replaced by v_1.

p.6, line 7 of Def. 1.2.1: “non-abelian” should be “non-additive”

p.8 diagram: the upper right entry should be $C_{p+1,q+1}$, not $C_{p+1,p+1}$.

p.12 line 1: $B \to C$ should be $B \xrightarrow{p} C$

p.12 line 9: “so is $\text{coker}(f) \to \text{coker}(g)$” should be “so is $\text{coker}(g) \to \text{coker}(h)$”

p.13 line -1: $Z_{n-1}(b)$ should be $Z_{n-1}(B)$

p.15 line 11: $B(-1)$ should be $B[-1]$

p.18 line 3: Replace the sentence “Give an example...” with: “Conversely, if C and $H_*(C)$ are chain homotopy equivalent, show that C is split.”

p.18 line 18: Replace $i = 1, 2$ with $i = 0, 1$

p.21 Ex.1.5.3: Add extra paragraph: If $f : B \to C$, $g : C \to D$ and $e : B \to C$ are chain maps, show that e and gf are chain homotopic if and only if there is a chain map $\gamma = (e, s, g)$ from $\text{cyl}(f)$ to D. Note that e and g factor through γ.

p.24 line -7: ∂ should be α

p.26 line -10: { let} $C^\infty(U)$ be ... that C^∞ is a sheaf...

p.27 line 7: the contour integral should be $\frac{1}{2\pi i} \oint f'(z)dz/f(z)$, not $\frac{1}{2\pi i} \oint f(z)dz$.

p.29 line 17: should read “[Freyd, p. 106], every small full abelian subcategory of $\mathcal{L}$ is equivalent to a full abelian subcategory of the category R-**mod** of modules over the ring”

p.32 line 1: 2.6.3 should be 2.6.4

p.33 line -2: after “no projective objects” add “except 0.”

p.34 line -14 (Ex. 2.2.1): Add this sentence before the hint: “Their brutal truncations $\sigma_{\geq 0}P$ form the projective objects in $\mathbf{Ch}_{\geq 0}$.”

p.35 line 8: replace “chain map” by “quasi-isomorphism”

p.37 line 1: delete ’commutative’

p.44 line 11: ’gf’ should be ’gf’ (math font)

p.44 line -9: $L_i(A)$ should be $L_iF(A)$

p.40 line -8: the map F should be f

p.47 line -6: the m^{th} *syzygy*

p.49 line 1: $L_n(f)$ should be $L_nF(f)$

p.49 line -11 (Ex. 2.4.4): Replace “the mapping cone $\text{cone}(A)$ of exercise 1.5.1” by the following text: “$\sigma_{\geq 0}\text{cone}(A)[1]$, where $\text{cone}(A)$ is the mapping cone of exercise 1.5.1. If $\mathcal{A}$ has enough projectives, you may also use the projective objects in $\mathbf{Ch}_{\geq 0}(\mathcal{A})$, which are described in Ex. 2.2.1.”

p.50 line -10: $\text{Hom}_R(\ ,B)$-acyclic should be $\text{Hom}_R(A,\ )$-acyclic.

p.55 line -9 to -6: Replace paragraph with:
We say that $\mathcal{A}$ satisfies axiom (AB4) if it is cocomplete and direct sums of monics are monic, i.e., homology commutes with direct sums. This is true for **Ab** and **mod**-R. (Homology does not commute with arbitrary colimits; the derived functors of colim intervene via a spectral sequence.) Here are two consequences of axiom (AB4).

p.55 line -5: delete “cocomplete” and insert “satisfying (AB4)” before “has enough projectives”

p.56 line 13: (1) and (2) should be switched

p.57 lines 2,-10: a is the image (‘a’ should be ‘a’ twice) p.57 line 4: $a_{jk} \in A_j$ should be $a_j \in A_j$

p.58 lines 6–7: Replace the text "If...and" with: Suppose that $\mathcal{A} = R$-**mod** and $\mathcal{B} =$**Ab** (or $\mathcal{A}$ is any abelian category with enough projectives, and $\mathcal{A}$ and $\mathcal{B}$ satisfy axiom (AB5)). If"

p.58 line 9: $F(A)$ should be $F(A_i)$

pp.60–61: several 2-symbol subscripts are missing the comma (e.g., C_{pq} means $C_{p,q}$).

p.62 lines 8: Replace the sentence "Finally...acyclic." with: "Show that $\mathrm{Tot}^{\oplus}(D)$ is not acyclic either."

p.66 line 9: $pB = 0$ should be $pb = 0$.

p.67 line 5: Tor_* should be Tor_1

p.70 end of line -11: $j =$ should be $i =$

p.74 Exercise 3.3.1: $\cdots \cong \mathbb{Z}_{p^\infty}$ should be $\cdots \cong (\mathbb{Q}/\mathbb{Z}[1/p]) \times \hat{\mathbb{Q}}_p/\mathbb{Q}$.

p.74 Exercise 3.3.5: In the display, replace A/pA with A^*/pA^* and delete the final '$= 0$'. On the next line (line -1), 'A is divisible' should be 'A^* is divisible, i.e., A is torsionfree'.

p.82 line 6: ... axiom $(AB5*)$ (filtered limits are exact), the above proof can be modified to show ...

p.82 line 9: add sentence: Neeman has given examples of abelian categories with $(AB4*)$ in which Lemma 3.5.3 and Corollary 3.5.4 both fail; see *Invent. Math.* 148 (2002), 397–420.

p.82 line -8: 'complete' should be 'complete and Hausdorff'

p.85 line -4: Then $\mathrm{Tot}(C) = \mathrm{Tot}^{\Pi}(C)$ is ...

p.86 line 2: "nonzero columns." (not rows)

p.89 line 8: $\big\} \otimes \big\{$ should be $\big\} \oplus \big\{$.

p.90 line 8: $\prod_{p+q\quad n-1}$ should be $\prod_{p+q=n-1}$

p.93 line -3: 'all R-modules B' should be 'all R-modules A'

p.95 line 17: 'the' (before $pd_R(P)$) should be 'then'

p.97 line 14: Add sentence:
"If in addition R is finite-dimensional over a field then R is quasi-Frobenius $\Leftrightarrow$ R is Frobenius."

p.101 line -5: $n < \infty$ should be $d < \infty$

p.102 lines 3,4: $\leq 1 + n$ should be $\leq 1 + d$ twice

p.113 line 13: the final $H_q(C)$ should be $H_{q-1}(C)$

p.122 line 9: $-(r+1)/r$ should be $-(r-1)/r$

p.124 line 7: E^∞_{0n} is a quotient of E^a_{0n} and each E^∞_{n0} is a subobject of E^a_{n0}.

p.124 line -7: $0 \to E^2_{0n} \to H_n \to E^2_{1,n-1} \to 0$.

p.127 line 14 (**): $(-1)^{p_1}$ should be $(-1)^{p_1+q_1}$

p.131 lines 3–4: $SO(1)$ should be $SO(2)$ twice

p.131 line 7: Replace "$H_2(SO(3)) \cong \mathbb{Z}$, ...isomorphism." with "d^2 is an injection, and $H_2(SO(3)) = 0$."

p.132 line -3: $F_sH_n(C)$ should be F_sC_n

p.134 line 8: The filtration on the complex C' is bounded below, the one on C'' ...

p.135 line 12: the superscripts r should be $r+1$, viz.,

$$\cdots E^{r+1}_{p+r}(\mathrm{cone}\, f) \to E^{r+1}_p(B) \to E^{r+1}_p(C) \to E^{r+1}_p(\mathrm{cone}\, f) \to E^{r+1}_{p-r}(B) \cdots .$$

p.135 line 18: Insert after $d(c) = 0$: "(This assumes that (AB5) holds in $\mathcal{A}$.)"

p.135 line -13: after 'exhaustive' add: "and that $\mathcal{A}$ satisfies (AB5)."

p.135 line -4: replace text starting with E^1_{p0} to read: E^1_{p0} is $\bar{C}_p = C_p/(F_{p-1}C_p + d(F_pC_{p+1})$; $\bar{C}$ is the top quotient chain complex of C, and $d^1_{p0} : E^1_{p0} \to E^1_{p-1,0}$ is induced from $d : C_p \to C_{p-1}$.

p.136 line -9,-8: let $F_{-p}C$ be 2^pC $(p \geq 0)$.

p.136 line -7: "Each row" should be "Each column" p.137 Cor. 5.5.6 should read: If the spectral sequence weakly converges, then the filtrations on $H_*(C)$ and $H_*(\widehat{C})$ have the same completions.

p.143 line 15: $H_q(A)$ should be $H_q(Q)$

p.152 line 8: $\xrightarrow{\otimes_S R}$ should read $\xrightarrow{\otimes_R S}$.

p.154 line -2: $\mathcal{E} = \mathcal{E}^{a+1}$ and $\mathcal{E}^r$ denotes the $(r-a-1)^{st}$ derived couple

p.154 line -1: $j^{(r)}$ has bidegree $(1-r, r-1)$

p.155 line 2: remove $\xrightarrow{i} D$ from diagram to read: $E^r_{pq} \xrightarrow{k} D^r_{p-1,q} \xrightarrow{j^{(r)}} E^r_{p-r,q+r-1}$.

p.155 line 6: starting with E^{a+1}.

p.160 display on line -7: delete ' and a in A'

p.168 line -1: $H_{1-n}(G;A)$ should be $H_{-1-n}(G;A)$

p.177 line 13: If m is odd, every automorphism of D_m stabilizing C_m is inner.

p.185 fourth line of proof of Classification Theorem: $\beta(1)$ should be $\beta(1) = 1$

p.186 line -3: $b_h g$ should be $b_h h$.

p.191 Cor. 6.7.9: ... of G on H induces an action of G/H on $H_*(H;\mathbb{Z})$ and $H^*(H;\mathbb{Z})$.

p.191 line -3: complex of (space missing)

p.193 line 19: delete '$\beta\sigma = 0$' so it reads '$(\sigma^2 = 0)$'

p.193 lines -3, -8; and p.194 line 7: "cocommutative" should be "coassociative"

p.196 line -4: If H is in the center of G and A is a trivial G-module then G/H acts trivially ...

p.201 Exercise 6.9.2: If ... and ... are central extensions, and X is perfect, show ...

p.203 lines 1–2: When $\mathbb{F}_q$ is a finite field, and $(n,q) \neq (2,2),(2,3),(2,4),(2,9),(3,2),(3,4),(4,2)$, we know that $H_2(SL_n(\mathbb{F}_q);\mathbb{Z}) = 0$ [Suz, 2.9]. With these exceptions, it follows that

p.213 Exercise 6.11.11: ... Show that for $i \neq 0$:

$$H^i(G;\mathbb{Z}) = \begin{cases} \mathbb{Z}_{p^\infty} & i = 2 \\ 0 & \text{else.} \end{cases}$$

p.226: Line 3 of Exercise 7.3.5 should read: δ-functors (assuming that that k is a field, or that N is a projective k-module):

p.238 lines 4–7: Replace these two sentences (Show that...it suffices to show that ...= 0.) by:

Conversely, suppose that $\mathfrak{g} = \mathfrak{f}/\mathfrak{r}$ for some free Lie algebra $\mathfrak{f}$ with $\mathfrak{r} \subseteq [\mathfrak{f},\mathfrak{f}]$, and $\mathfrak{g}$ is free as a k-module. Show that if $H^2(\mathfrak{g}, M) = 0$ for all $\mathfrak{g}$-modules M then $\mathfrak{g}$ is a free Lie algebra. *Hint:* It suffices to show that ...= 0.

p.256 line 8: identity (not identify)

p.257 line 15 (display): $\alpha_*(t)$ should be $\alpha_*(s)$

p.258 lines 1, 20 and -7: 'combinational' should be 'combinatorial'

p.261 lines -9,-8,-6: 'combinatorial' is misspelled three more times

p.262 line 18: $g_r = g(\sigma_r u)^{-1}$

p.262 line -13: 'every n' should be 'every sufficiently large n'

p.262 line -6: $\partial_i(y)$ should be $\partial_i(y) = x_i$

p.265 add to end of Exercise 8.3.3:

Extend exercise 8.2.5 to show that a homomorphism of simplicial groups $G \to G''$ is a Kan fibration if and only if the induced maps $N_n G \to N_n G''$ are onto for all $n > 0$. In this case there is also a long exact sequence, ending in $\pi_0(G'')$.

p.266 line 14: that $\sigma_i(x_i) \neq 0$, then $y = y - \sigma_i\partial_i y = \sum_{j>i} \sigma_j(x'_j)$. By induction, $y = 0$. Hence $D_n \cap N_n = 0$.

p.267 line 5: fix subscript on sum: $d\sigma_p(x) = \sum_{p+2}^n$

p.267 line 6: $d\sigma_p^2(x) + \sigma_p d\sigma_p(x) = \sum_{i=p+2}^{n+1} \cdots + \sum_{i=p+2}^{n}$

p.267 line 7: $= (-1)^p \sigma_p(x)$.

p.267 line 8: Hence $\{s_n = (-1)^p \sigma^p\}$

p.268 line -3: '$\{0, 1, ..., i-1\}$' should be '$\{0, 1, ..., i\}$'

p.278 display on line 8: 1 should be subtracted from the subscripts: $\sigma^h_{\mu(n)-1} \cdots \sigma^h_{\mu(p+1)-1} \sigma^v_{\mu(p)-1} \cdots \sigma^v_{\mu(1)-1}$

p.280 lines 10–11: $\eta\colon 1_{\mathcal{C}} \to UF$ and ... $\varepsilon\colon FU \to 1_{\mathcal{B}}$. (switch $\mathcal{B}$ and $\mathcal{C}$)

p.283 line -2: $\bar{r}_i \bar{r}_{i+1}$ should be $\overline{r_i r_{i+1}}$

p.287 lines -5, -4: "is an exact sequence" should be "is a sequence" and "is also exact" should be "is exact"

p.290 line before 8.7.9: Insert sentence: The proof of Theorem 3.4.3 goes through to prove that $\mathrm{Ext}^1_{R/k}(M, N)$ classifies equivalence classes of k-split extensions of M by N.

p.291 line -2: ... to $(R/I)^d$. If each $x_i R \subset R$ is k-split then:

p.294 line -8: If M is an R-module, ('a k' should be 'an R')

p.295 line -4 (display): $D^1(R, M)$ should be $D^1(R/k, M)$

p.296 8.8.6: "If k is a field" should be "If R is a field"

p.297, line -9: the sequence should read

$$\cdots \to D_{n+1}(R/K, M) \to D_n(K/k, M) \to D_n(R/k, M) \to D_n(R/K, M) \to D_{n-1}(K/k, M) \to \cdots.$$

p.298 line 2 of 8.8.7: *Commalg* should be in roman font: **Commalg**

p.301 lines 4–5: the ranges should be "if $0 < i \le n$" and "if $i = n+1$" respectively.

p.304 Exercise 9.1.3: the variable n should m each time (y_n, R^n, n, $p < n$); and x (on line 17) should be $\mathbf{x}$

p.307 line -1 should read:

As $\mathrm{Tor}_1^{R^e/k}(R^e, M) = 0$, the long exact relative Tor sequence (Lemma 8.7.8) yields

p.322: On line 1, insert "If $1/2 \in k$," before "$\Omega^*_{R/k}$ is the free graded-commutative" and (on line 4) add the sentence: In general, $\Omega^*_{R/k}$ is the free alternating R-algebra generated by $\Omega^1_{R/k}$.

p.354 line -6: Add sentence: It also follows from the Connes-Karoubi theorem on noncommutative de Rham homology in C.R. Acad. Sci. Paris, t. 297 (1983), p. 381–384.

p.359 Exercise 9.9.5: This is wrong; replace it with:

Exercise 9.9.5 (Grauert-Kerner) Consider the artinian algebra $R = k[x, y]/(\partial f/\partial x, \partial f/\partial y, x^5)$, where $f = x^4 + x^2y^3 + y^5$. Show that $I = (x, y)R$ is nilpotent, and f is a nonzero element of $H^0_{dR}(R)$ which vanishes in $H^0_{dR}(R/I)$.

p.370 line -6: [ho-]"mopy" should be [ho-]"motopy" and b,″ should be b'',

p.376 line -10: diagram ... commutes up to chain homotopy.

p.381 line 6: '(left)' should be '(right)'

p.382 lines-6,-7 (10.3.8): the two occurrences of 'right' should be 'left'

p.384 lines 9, 11: 'left' fraction should be 'right' fraction

p.384 line 13: Replacing 'right' by 'left'

p.385 line 1: $\mathcal{B}$ is a small category and $\mathrm{Ext}(A, B)$ is a set for all $A \in \mathcal{A}$, $B \in \mathcal{B}$. Then show that...

p.386 lines 7–9: six occurrences of g should be v: ...there should be a $v : X \to Z$... $f - g = uv$. Embed v in an exact triangle (t, v, w) ... Since $vt = 0$, $(f - g)t = uvt = 0$, ...

p.386 lines 14–18: Replace the two sentences "Given us_1^{-1} : ... triangle in $\mathbf{K}$" with: "The exact triangles in $S^{-1}\mathbf{K}$ are defined to be those triangles which are isomorphic, in the sense of (TR1), to the image under $\mathbf{K} \to S^{-1}\mathbf{K}$ of an exact triangle in $\mathbf{K}$."

p.386 line 20: replace "straightforward but lengthy; one uses the fact" with "straightforward; one uses (TR3) and the fact that..."

p.387 line -12 (10.4.5): delete 'well-powered' (Gabber points out that this condition is superfluous).

p.400 line 8: the last two (A, B) should be $(A, T^n B)$

p.400 line -7: $\mathrm{Hom}(-, B)$ should be $\mathrm{Hom}^{\cdot}(-, B)$

p.405 line -2: a natural homomorphism in $\mathbf{D}(R)$, which is an isomorphism if either each C_i is fin. gen. projective or else A is quasi-isomorphic to a bounded below chain complex of fin. gen. projective R-modules:

p.420 line 13: in exercise 6.11.3 (not 6.11.4)

p.427 line -1: $I \in I$ should be $i \in I$

p.428 line 16: $F_i \to F_i \to C$ should be $F_j \to F_i \to C$

p.431 line 6: 'functions' should be 'functors.'

p.435 under 'AB4 axiom': add page 55

p.439 add entry to Index under 'double chain complex':
Connes' — $\mathcal{B}$. *See* Connes' double complex.

p.444, under 'Lie group': [page] 158 should be 159

p.445, line -6: 'Øre' should be 'Ore'

p.448 column 2: lines 29-30 should only be singly indented ("— of" refers to spectral sequence)

References

[EM] Eilenberg, S., and Moore, J. "Limits and Spectral Sequences." *Topology* **1** (1961): 1–23.