MATHEMATICS COMPETITION
UNIVERSITY OF MARYLAND
1997 PART II

Wednesday, December 3, 1997, 1 p.m.–3 p.m.

Use SEPARATE sheets for different problems. Submit only the material you want graded. Each problem is worth 30 points. No calculators are allowed. Proofs must be given for all answers.

1. Prove that for every point inside a regular polygon, the average of the distances to the sides equals the radius of the inscribed circle. The distance to a side means the shortest distance from the point to the line obtained by extending the side.

2. Suppose we are given positive integers $a_1, a_2, \ldots, a_{1997}$ (not necessarily distinct). Show that it is possible to choose some numbers from this list such that their sum is a multiple of 1997.

3. You have Blue blocks, Green blocks and Red blocks. Blue blocks and green blocks are 2 inches thick. Red blocks are 1 inch thick. In how many ways can you stack the blocks into a vertical column that is exactly 12 inches high? (For example, for height 3 there are 5 ways: RRR, RG, GR, RB, BR.)

4. There are 1997 nonzero real numbers written on the blackboard. An operation consists of choosing any two of these numbers, a and b, erasing them, and writing $a + \frac{b}{2}$ and $b - \frac{a}{2}$ instead of them. Prove that if a sequence of such operations is performed, one can never end up with the initial collection of numbers.

5. An $m \times n$ checkerboard (m and n are positive integers) is covered by nonoverlapping tiles of sizes 2×2 and 1×4. One 2×2 tile is removed and replaced by a 1×4 tile. Is it possible to rearrange the tiles so that they cover the checkerboard?