1. By assumption $I = 365 \times 4 = 1460$, $S = 50 \times 52 = 2600$ and $C = 180 \times 8 = 1440$. The answer is (c).

2. If we pair up the numbers we obtain:

$$(1 - 2) + (3 - 4) + \cdots + (2015 - 2016) + 2017 = (-1) \times 1008 + 2017 = 1009$$

The answer is (a).

3. Substituting $y = 2x + 1$ into the equation of the circle, we get $x^2 + (2x + 1)^2 = 13$. Simplifying we get $5x^2 + 4x - 12 = 0$. Factoring we get $(5x - 6)(x + 2) = 0$. Therefore $x = -2, 6/5$. The answer is (b).

**Note:** Since you know $x = -2$ is a solution you can find the other solution using the fact that the product of the two solution is $-12/5$. This is one of the so-called Vieta’s Formulas.

4. We know there are 10 candies that contain neither marshmallow nor chocolate. We also know that there are 75 that contain chocolate. Therefore, there are $100 - 75 - 10 = 15$ candies that contain marshmallow but no chocolate. The answer is (a).

5. Suppose $s$ is the speed of Zeno. The time that it would take Zeno to finish the race is $10/s$.

The speed of Achilles in the first 5 miles of the race is $2s$ and in the second 5 miles it is $s/2$. Therefore, the time that it takes Achilles to finish the race is $\frac{5}{2s} + \frac{5}{s/2}$.

By assumption, we have $\frac{10}{s} + 1 = \frac{5}{2s} + \frac{5}{s/2} = \frac{5}{2s} + \frac{10}{s}$. Thus $\frac{5}{2s} = 1$, which implies $s = 2.5$. The answer is (b).

6. By properties of log, $a^{\log_a b} = b$. Therefore $2^{\log_2(3)} + 3^{\log_3(4)} + 4^{\log_4(5)} = 3 + 4 + 5 = 12$. The answer is (e).

7. Let $s$ be the speed of each train. In 6 minutes a train from Baltimore to Washington travels half the distance between two trains that travel from Washington to Baltimore. Therefore $w = 6 \times 2 = 12$. The answer is (d).

8. Suppose $x$ is the average consumption of all 10 children. Then the oldest child consumes $18 + x$.

By assumption we have $\frac{(18 + x) + 15 \times 9}{10} = x$. Therefore, $18 + x + 15 \times 9 = 10x$, which implies $x = 17$. The oldest child eats $18 + 17 = 35$ pieces of candy. The answer is (e).

9. Positive divisors of 100 are 1, 2, 4, 5, 10, 20, 25, 50 and 100. The product is $10^9$. The answer is (b).

10. Note that $\cos(180 - x) = -\cos x$. Using this identity we see that $\cos 1^\circ = -\cos 179^\circ, \ldots, \cos 89^\circ = -\cos 91^\circ$. The answer is (a).
11. Writing several terms of the sequence we see the terms repeat after the 6th term:

\[3, 7, 4, -3, -7, -4, 3, 7, \ldots\]

Since the remainder of 2017 when divided by 6 is 1, we have \(a_{2017} = a_1 = 3\). The answer is (d).

12. **Geometric solution.** The inequality \(|k - 50| < |k - 200|\) implies \(k\) is closer to 50 than to 200. This means \(k\) is less than \(\frac{50 + 200}{2} = 125\). Similarly, the inequality \(|k - 200| < |k - 10|\) implies \(k\) is closer to 200 than to 10. This means \(k\) is more than \(\frac{10 + 200}{2} = 105\). Thus \(105 < k < 125\). The number of such integers is \(125 - 105 - 1 = 19\).

**Algebraic solution.** Squaring we get \(k^2 - 100k + 2500 < k^2 - 400k + 40000 < k^2 - 20k + 100\). Therefore \(300k < 37500\) and \(380k > 39900\). Therefore \(105 < k < 125\).

The answer is (c).

13. Let \(0 < k < 2000\) be odd. We claim that precisely one integer of form \(2^r k\) appears in the list 1001, 1002, \ldots, 2000. If \(r\) is the largest integer satisfying \(2^r k \leq 1000\), then \(1000 < 2^{r+1} k \leq 2000\) and that \(2000 < 2^{r+2} k\). This proves our claim. This means every positive odd integer less than 2000 appears once in the list \(p(1001), p(1002), \ldots, p(2000)\). Therefore the answer is \(1 + 3 + \cdots + 1999 = \frac{1 + 1999}{2} \times 1000 = 1000000\). The answer is (e).

14. Let \(a_n\) be the number of ways a \(2 \times n\) rectangle can be tiled with 1 \(\times 2\) tiles. Clearly \(a_1 = 1\) and \(a_2 = 2\). To get a tiling for a \(2 \times n\) rectangle we can either tile a \(2 \times (n - 1)\) rectangle and add a \(1 \times 2\) rectangle or tile a \(2 \times (n - 2)\) rectangle and add two \(1 \times 2\) rectangles. Thus, \(a_n = a_{n-1} + a_{n-2}\). Therefore the sequence \(a_n\) is 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 The answer is (e).

15. Suppose \(X \cap Y\) has \(n\) elements. Since \(X\) and \(Y\) have 100 elements, \(X \cup Y\) has 200 - \(n\) elements. Let \(X = \{x_1, \ldots, x_{100}\}\) and \(Y = \{y_1, \ldots, y_{100}\}\). By assumption \(x_1 + \cdots + x_{100} + y_1 + \cdots + y_{100} = 0\). This sum has all elements of \(X \cup Y\), with the ones in \(X \cap Y\) each appearing twice. By assumption, the sum of elements of \(X \cap Y\) is \(-4 \cdot n\) and the sum of elements of \(X \cup Y\) is \(1 \cdot (200 - n)\). Therefore \(-4n + 200 - n = 0\). Therefore \(n = 40\). The answer is (c).

16. By assumption \(ABC\) is an isosceles triangle. Let \(x = \angle BAC = \angle BCA\). Note that \(\angle ABC = 180 - 2x\). By extended law of sines we have \(\frac{1}{\sin x} = \frac{|AC|}{\sin(180 - 2x)} = 2\). This implies \(\sin x = 0.6\), thus \(\cos x = 0.8\). Therefore, \(\sin(180 - 2x) = \sin(2x) = 2 \sin x \cos x = 2 \times 0.6 \times 0.8 = 0.96\). This implies \(\frac{|AC|}{0.96} = 2\), therefore \(|AC| = 1.92\). The answer is (d).

17. Let \(x\) be the side length of the square. From \(B\), we draw a line perpendicular to \(\ell_1\) and \(\ell_3\). This creates two right triangles with \(AB\) and \(BC\) as their hypotenuses. The two right triangles are congruent. Therefore \(x^2 = 5^2 + 7^2 = 74\). The answer is (c).

18. By assumption, \(x^2 \leq 1\), which implies \(|x| \leq 1\). This implies \(x^4 \leq x^2\); similar for \(y, z\) and \(w\). Therefore \(x^4 + y^4 + z^4 + w^4 \leq x^2 + y^2 + z^2 + w^2\). Since the equality holds, we have \(x^2 = x^4\), which implies \(x^2 = 0, 1\); similar for \(y^2, z^2\) and \(w^2\). Since \(x^2 + y^2 + z^2 + w^2 = 1\), one of \(x^2, y^2, z^2, w^2\) is one and the others are zero. Thus the maximum of \(x + y + z + w\) is 1. The answer is (a).
19. Suppose \( ABC \) is the triangle formed by the three tangent lines. Let \( P \) and \( Q \) be points of tangency of \( AB \) with the circles, so that \( P \) is between \( A \) and \( Q \). Let \( O_1 \) and \( O_2 \) be the centers of circles closer to \( A \) and \( B \), respectively. We know \( |PQ| = |O_1O_2| = 2 \). Since \( \angle O_1AP = 30^\circ \), \( AP = \sqrt{3} \). Thus, \( AB = 2 + 2\sqrt{3} \). This implies the area of \( ABC \) is \( \frac{\sqrt{3}}{4} (2 + 2\sqrt{3})^2 = \sqrt{3}(4 + 2\sqrt{3}) = 4\sqrt{3} + 6 \). The answer is (a).

20. Not that in general, given real numbers \( S \), there are real numbers \( a \) and \( b \) for which \( a + b = S \) and \( ab = P \), if the equation \( x^2 - Sx + P = 0 \) has two real roots, which is true iff the discriminant is non-negative. In other words \( S^2 - 4P \geq 0 \). Therefore \( (2c - 2)^2 - 4(2c^2 + c - 3) \geq 0 \) \( \Rightarrow 4c^2 - 8c + 4 - 8c^2 - 4c + 12 = -4c^2 - 12c + 16 \geq 0 \) \( \Rightarrow c^2 + 3c - 4 \leq 0 \)

Therefore, \( -4 \leq c \leq 1 \). The answer is (a).

21. **First Solution.** We use trigonometry. Let \( x = \angle DBA = \angle DBC \). Thus, \( \angle ABC = \angle ACB = 2x \), which implies \( \angle BDC = 180 - 3x \) and \( \angle BAC = 180 - 4x \). Using the Law of Sines in triangles \( ABD \) and \( BDC \), we obtain \( \frac{|AD|}{|BD|} = \frac{\sin x}{\sin(180 - 4x)} \) and \( \frac{|BC|}{|BD|} = \frac{\sin(180 - 3x)}{\sin(2x)} \). By assumption \( \frac{|AD|}{|BD|} + 1 = \frac{|BC|}{|BD|} \). Using the identities that we found above we conclude,

\[
\frac{\sin x}{\sin(180 - 4x)} + 1 = \frac{\sin(180 - 3x)}{\sin(2x)} \Rightarrow \frac{\sin x + \sin(4x)}{\sin(4x)} = \frac{\sin(3x)}{\sin(2x)} \Rightarrow \frac{\sin x + \sin(4x)}{2\sin(2x)\cos(2x)} = \frac{\sin(3x)}{\sin(2x)}.
\]

Multiplying by \( \sin(2x) \) and using the formula \( 2\sin a \cos b = \sin(a + b) + \sin(a - b) \) we get the following:

\[
\sin x + \sin(4x) = 2\cos(2x)\sin(3x) = \sin(5x) + \sin(x) \Rightarrow \sin(4x) = \sin(5x).
\]

Therefore \( 4x + 5x = 180 \Rightarrow x = 20 \). This implies \( \angle BAC = 180 - 4 \times 20 = 100 \).

**Second Solution.** This is a less computational but more clever solution. Select point \( E \) on side \( BC \) such that \( |CE| = |AD| \). By assumption, \( |BD| = |BE| \). By Angel-Bisector Theorem, \( \frac{|AB|}{|BC|} = \frac{|AD|}{|DC|} = \frac{|CE|}{|CD|} \). Therefore the triangle \( ABC \) and \( ECD \) are similar by SAS, which implies \( \angle EDC = \angle DCE \). Similar to the first solution let \( x = \angle ABD \). We see that \( \angle CDE = 2x \), since \( DEC \) is isosceles. \( \angle ADE = 90 - x/2 \), since \( BDE \) is isosceles, and \( \angle ADB = 3x \). Therefore \( 3x + 90 - x/2 + 2x = 180 \). Hence, \( x = 20 \). Similar to the first solution we obtain \( \angle BAC = 100 \).

The answer is (a).

22. Note that if \( x < y \), then \( x^3 < y^3 \), therefore \( f \) is increasing. Note also that \( f(-8) = -552 \), which implies \( -8 < f(x) < 4x \). Therefore \( 0 < x^3 + 4x \) and \( x^3 < 8 \). Hence, \( 0 < x < 2 \). The answer is (b).

23. We see that \( f(0) = c \), \( f(1) = a + b + c \) and \( f(1/2) = a/4 + b/2 + c \). Solving this system we obtain that \( 2a + b = f(0) + 3f(1) - 4f(1/2) \). Thus, \( 2a + b \leq 8 \). Equality holds for \( f(x) = 8x^2 - 8x + 1 \). The answer is (e).
24. For every $x$ we have, $f(1-x) = \frac{4^{1-x}}{2+4^{1-x}} = \frac{4}{2 \cdot 4^x + 4} = \frac{2}{4^x + 2}$. This shows that $f(x) + f(1-x) = 1$. Therefore,

$$f(\frac{1}{14}) + f(\frac{13}{14}) = f(\frac{2}{14}) + f(\frac{12}{14}) = \cdots = f(\frac{6}{14}) + f(\frac{8}{14}) = 2f(\frac{7}{14}) = 1.$$  

The answer is (c).

25. **First solution.** Let $f(x)$ be the given polynomial. Note that if $x$ is a solution, so is $\frac{2}{x}$. This can be verified by substituting $\frac{2}{x}$ for $x$. We also see that $f(0) > 0$ and $f(-1) < 0$, therefore the graph of $f$ crosses the $x$-axis at least once between $x = 0$ and $x = -1$. Let $r$ be such a root of $f$ between $-1$ and $0$. We know $\frac{2}{r} < -2$ and that $\frac{2}{r}$ is another root of $f(x)$. Similarly $f$ has a root between $0$ and $1$ and one larger than $2$. By the discussion above $r_1r_2 = r_3r_4 = 2$. Therefore $r_1r_2 + r_3r_4 = 4$.

**Second Solution.** Similar to above, solutions come in pairs $(r, 2/r)$. This suggests substituting $S = x + \frac{2}{x}$. This implies $S^3 = x^3 + \frac{8}{x^3} + 6x + \frac{12}{x} = x^3 + \frac{8}{x^3} + 6S$. Dividing the equation by $x^3$, we obtain

$$x^3 + \frac{8}{x^3} - 15(x + \frac{2}{x}) + 20 = 0 \Rightarrow S^3 - 6S - 15S + 20 = 0 \Rightarrow (S-1)(S-4)(S+5) = 0$$

Therefore $S = 1, 4, -5$. Solving each we get four real solutions $x = 2 \pm \sqrt{2}, \frac{-5 \pm \sqrt{17}}{2}$. The answer is (e).