29th ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
November 28, 2007, 1:00–3:00

NO CALCULATORS
2 hours

1. One hundred hobbits sit in a circle. The hobbits realize that whenever a hobbit and his two neighbors add up their total rubles, the sum is always 2007. Prove that each hobbit has 669 rubles.

2. There was a young lady named Chris,
   Who, when asked her age, answered this:
   "Two thirds of its square
   Is a cube, I declare."
   Now what was the age of the miss?

   (a) Find the smallest possible age for Chris. You must justify your answer. (*Note:* ages are positive integers; "cube" means the cube of a positive integer.)
   (b) Find the second smallest possible age for Chris. You must justify your answer. (Ignore the word "young.")

3. Show that
$$\sum_{n=1}^{2007} \frac{1}{n^3+3n^2+2n} < \frac{1}{4}.$$

4. (a) Show that a triangle ABC is isosceles if and only if there are two distinct points P_1 and P_2 on side BC such that the sum of the distances from P_1 to the sides AB and AC equals the sum of the distances from P_2 to the sides AB and AC.
   (b) A convex quadrilateral is such that the sum of the distances of any interior point to its four sides is constant. Prove that the quadrilateral is a parallelogram. (*Note:* "distance to a side" means the shortest distance to the line obtained by extending the side.)

5. Each point in the plane is colored either red or green. Let ABC be a fixed triangle. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.