1. Suppose the contents of the envelopes in dollars are \( x, x + 1, \ldots, x + 5 \). The total money in the envelopes, thus, is \( 6x + 15 \). The total amount in the remaining envelopes is between \( x + x + 1 + \ldots + x + 4 = 5x + 10 \) and \( x + 1 + \ldots + x + 5 = 5x + 15 \). Thus, \( 5x + 10 \leq 2018 \leq 5x + 15 \), which implies \( x + 2 \leq 2018/5 \leq x + 3 \). This shows \( x = 401 \). Since \( 2018 = 5 \times 401 + 13 \), the last envelope contains \( x + 2 = 403 \) dollars. The answer is 403.

2. Let \( O \) be the center of the circle. Note that \( A \) and \( O \) are both on the perpendicular bisector of \( BC \). Thus \( AO \perp BC \). Since \( DE \) and \( BC \) are parallel and \( AO \perp BC \), we have \( AO \perp DE \).

Let \( r \) be the radius of the circle and \( M \) be the point of intersection of \( DE \) and \( AO \). We need to show \( |MO| > r \). Since triangles \( AOB \) and \( ADM \) are similar, we have

\[
\frac{|AO|}{|AD|} = \frac{|AB|}{|AM|}
\]

Since \( |AD| = |AB|/2 \), we have

\[
|AM| = \frac{|AB|^2}{2|AO|} = \frac{|AO|^2 - r^2}{2|AO|}
\]

This shows \( |MO| = |AO| - |AM| = \frac{|AO|^2 + r^2}{2|AO|} \). To show \( |MO| > r \), it is enough to show \( |AO|^2 + r^2 > 2r|AO| \), which is equivalent to \( (|AO| - r)^2 > 0 \). This holds since \( |AO| > r \).

3. Let \( T = \{x + 1 \mid x \in S \} \subseteq \{1, \ldots, n - 1\} \). By assumption, \( S \) is closed if and only if \( T \) satisfies the following

(a) \( 1 \in T \).
(b) If \( x \in T \), then \( n - x \in T \).
(c) If \( x \in T \), \( y \geq 1 \) and \( y \) divides \( x \), then \( y \in T \).

Suppose \( n \) is prime and \( S \) is closed. We will show \( T = \{1, \ldots, n - 1\} \). On the contrary assume \( k \) is the smallest positive integer less than or equal to \( n - 1 \) that is not in \( T \). Note that \( k \geq 2 \) and thus \( k \) does not divide \( n \). By division algorithm, there are integers \( q \) and \( r \) for which \( n = kq + r \) and \( 0 < r < k \). By the choice of \( k \), we have \( r \in T \) and thus by (b), \( n - r \in T \). This implies \( kq \in T \). By (c), \( k \in T \), which is a contradiction. This shows \( T = \{1, \ldots, n - 1\} \).

Now assume \( n \) is composite and let \( T \) be the set of all positive integers less than or equal to \( n - 1 \) that are relatively prime to \( n \). Clearly (a) is satisfied. If \( x \in T \), then \( n - x \) is also relatively prime to \( n \) and thus \( n - x \in T \), which shows (b) is satisfied. If \( x \) is relatively prime to \( n \), then every divisor of \( x \) is also relatively prime to \( n \), which shows (c) is satisfied. Also, if \( 1 < r < n \) is a divisor of \( n \), then \( r \notin T \), which means \( T \neq \{1, \ldots, n - 1\} \). This completes the proof.

4. For any \( n \), let \( a_n, c_n \), and \( i_n \) denote the total number of knight paths of length \( n \) which begin at \( A \) and end at \( A, C, \) and \( I \), respectively. We are trying to fine \( a_{2018} - i_{2018} \).

First we evaluate \( a_2 \) and \( i_2 \). There are two knight paths from \( A \) to \( A \) (specifically, \( AHA \) and \( AFA \)), one path from \( A \) to \( C \) (\( AHIC \)), one path from \( A \) to \( G \) (\( AFG \)) and no paths from \( A \) to \( I \). Thus \( a_2 = 2 \) and \( i_2 = 0 \).

Since there are 2 knight paths of length 2 from \( A \) to \( A \) and one each from \( C \) to \( A \) and \( G \) to \( A \), we have \( a_{n+2} = 2a_n + c_n + c_n = 2a_n + 2c_n \)
By similar reasoning, $i_{n+2} = 2i_n + c_n + c_n = 2i_n + 2c_n$.

Subtracting these two equations we obtain $a_{n+2} - i_{n+2} = 2(a_n - i_n)$. Therefore, $a_{2018} - i_{2018} = 2^{1008}(a_2 - i_2) = 2^{1009}$.

5. We claim this is impossible. Suppose on the contrary $n$ strips of widths $d_1, \ldots, d_n$ cover a unit disk $D$. Let $S$ be the sphere whose great circle is the boundary of $D$. For each strip draw two planes perpendicular to the plane containing $D$ through parallel lines that bound that strip. Let $R_1, \ldots, R_n$ be the regions that are created by these planes on the sphere $S$. Note that by assumption the sum of areas of $R_1, \ldots, R_n$ is at least the area of $S$.

Recall that the area of a spherical cap is $2\pi rh$, where $r$ is the radius of the sphere and $h$ is the height of the spherical cap. Since the area of each ring $R_i$ is the difference of the areas of two spherical caps, the area of $R_i$ is $2\pi \times \frac{1}{2} \times d_i$. This implies $\pi d_1 + \cdots + \pi d_n \geq 4\pi \frac{1}{2} = \pi$. Therefore, $d_1 + \cdots + d_n \geq 1$, which is a contradiction.