

Pointwise Ergodic Theorems for Group Representations in L^α

Abstract

Pointwise, maximal and dominated ergodic theorem with "weighted" averages for Lamperti representations of amenable σ -compact locally compact groups in L^α ($1 < \alpha < \infty$) over a σ -finite measure space (Ω, \mathcal{F}, m) will be considered; in particular, these theorems imply ergodic theorems for positive group representations and group actions; Cesaro averages represent a special case when the "weights" are non-zero constants on integrable sets. We discuss various conditions on the "weights" and their supports, under which these theorems hold. It is well-known that the pointwise ergodic theorem is not valid, in general, for Cesaro averages of powers of invertible positive operators in L^1 so our restriction to the case $\alpha > 1$ is quite natural (in the case when $m(\Omega) < \infty$, the pointwise ergodic theorem for "tempered" Cesaro averages of representations in $L^1(\Omega, \mathcal{F}, m)$, induced by measure preserving group actions, has been proved by a different method by E. Lindenstrauss).

The results generalize, in several directions, some of the results contained in the Ph.D. thesis by A. Shulman (Vilnius, 1988), in Shulman (1988) and in Tempelman (1992).

The presentation is based on a joint paper with Alexander Shulman.