

Prove: $\cos(36^\circ) - \cos(72^\circ) = \frac{1}{2}$

$$\cos(36^\circ) = \frac{e^{\frac{\pi}{5}i} + e^{-\frac{\pi}{5}i}}{2} \quad (1)$$

$$\cos(72^\circ) = \frac{e^{\frac{2\pi}{5}i} + e^{-\frac{2\pi}{5}i}}{2} \quad (2)$$

$$a = e^{\frac{\pi}{5}i} \quad (3)$$

With substitutions and multiplication by 2 what we must prove becomes:

$$a + \frac{1}{a} - \left(a^2 + \frac{1}{a^2} \right) = 1 \quad (4)$$

or

$$a^4 - a^3 + a^2 - a + 1 = 0 \quad (5)$$

But look at the angle of each coefficient in the last equation:

Power	Angle
1	0
$-a$	$\frac{6\pi}{5}$
a^2	$\frac{2\pi}{5}$
$-a^3$	$\frac{8\pi}{5}$
a^4	$\frac{4\pi}{5}$

The last equation consists of 5 points equally distributed equally in angle on the unit circle of the complex plane, thus their sum must be zero by symmetry.

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