

Homework #9 (due Monday, November 19)

Problem 1. Ch. 6, #22.

Problem 2. Let (x_1, \dots, x_n) be a sample from a population with pdf $f(x; \theta) = 3\theta x^2$, $-1 \leq x \leq 0$, $= 3(1 - \theta)x^2$, $0 \leq x \leq 1$ with θ , $0 < \theta < 1$ as a parameter.

- (i) Check that $f(x; \theta)$ is a pdf.
- (ii) Find the estimator $\tilde{\theta}$ of θ by the method of moments.
- (iii) Calculate $E(\tilde{\theta})$ and show that $\tilde{\theta}$ is an unbiased estimator of θ .
- (iv) For $x_1 = -0.8$, $x_2 = -0.3$, $x_3 = 0.1$, $x_4 = 0.7$ calculate the method of moments estimate of θ .

Problem 3. Let (x_1, \dots, x_n) be a sample from a population with pdf

$$f(x; \theta) = \frac{\theta - 1}{x^\theta}, \quad x \geq 1$$

with $\theta > 1$ as a parameter.

- (i) Check that $f(x; \theta)$ is a pdf.
- (ii) Write the likelihood function, then the loglikelihood and by differentiating the latter write the equation for finding the maximum likelihood equation (MLE).
- (iii) Find the MLE $\hat{\theta}$ of θ .
- (iv) For $x_1 = 1.3$, $x_2 = 1.8$, $x_3 = 2.0$, $x_4 = 1.2$, $x_5 = 2.3$ calculate the maximum likelihood estimate of θ .