MATH 410, HW 5

1. Compute the Taylor polynomial of degree 3 for:

$$f(x) = \sin(x) + x^{100},$$

at $x_0 = 0$.

2. Find the Taylor polymonial of degree 5 for:

$$f(x) = x^6 - 3x^4 + 2x - 1,$$

at $x_0 = 1$.

3. Find the Taylor series of:

$$f(x) = x^6 - 3x^4 + 2x - 1,$$

at $x_0 = -1$.

4. Let $f(x) = \tan(x)$, Using the fact that $f(0) = 0$ and $f'(x) = 1 + f^2(x)$, find the sum of the first six terms of the Taylor polynomial for f about 0.

5. Let

$$f(x) = e^{x^2} \int_0^x e^{-t^2} \, dt.$$

Prove that the Taylor series of f about 0 is equal to

$$\sum_{n=0}^{\infty} \frac{4^n n!}{(2n+1)!} x^{2n+1}.$$