MATH 411, HW 1

1. Let $\{u_k\} \subset \mathbb{R}^n$ be a sequence of points that converges to $u \in \mathbb{R}^n$. Show that the sequence of real numbers $\{\|u_k\|\}$ converges to $\|u\|$.

2. Prove that the union of a collection of open subsets of $\mathbb{R}^n$ is an open set in $\mathbb{R}^n$.

3. Show that sets which are simultaneously closed and open have empty boundary.

4. Prove that any closed ball in $\mathbb{R}^n$ is a closed subset of $\mathbb{R}^n$.

5. Let $U = \{(x, y, z) \in \mathbb{R}^3 : (x, y, z) \neq (0, 0, 0)\}$. Let $f : U \to \mathbb{R}$ be defined by

$$f(x, y, z) = \frac{y}{x^2 + y^2 + z^2}.$$

Prove that f is continuous on U.