MATH 411, HW 2

1. Let $\mathbf{x} \in \mathbb{R}^n$, and let $f : \mathbb{R}^n \to \mathbb{R}$ be such that $\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x})/\|\mathbf{x}\| = 0$. What can you say about the limit $\lim_{\mathbf{x}\to\mathbf{0}} f(\mathbf{x})$?

2. Analyze the following limits ($x, y \in \mathbb{R}$):

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^2+y^2}, \qquad \lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2}-1}{x^2+y^2}.$$

3. Let $f(x, y, x) = xyz + x\sin(yz) + x^y$ ($x, y, z \in \mathbb{R}$). Compute all partial derivatives of 1st and 2nd order of f.

4. Let $f : \mathcal{O} \to \mathbb{R}$, where $\mathcal{O}$ is an open subset of $\mathbb{R}^n$. Assume f is continuously differentiable. Does this imply that f is continuous on $\mathcal{O}$? Prove or disprove.

5. Solve Problem 13.3.10 in text.