MATH 411, HW 3

1. Suppose that the function $f : \mathbb{R}^2 \to \mathbb{R}$ has a tangent plane at the point $(x_0, y_0, f(x_0, y_0))$. What can you say about the limit

$$\lim_{(x,y)\to(x_0,y_0)} \frac{f(x,y) - f(x_0,y_0)}{(x-x_0)^2 + (y-y_0)^2}?$$

2. Does there exist a linear mapping $F : \mathbb{R}^3 \to \mathbb{R}^3$ such that $F(1,2,3) = (2,3,4)$ and $F(-1,-2,-3) = (3,2,1)$?

3. Consider the following mappings $\mathbb{R}^2 \to \mathbb{R}^2$:

$$f(x,y) = (e^x + y, \sin(2x)); g(x.y) = (2x + \cos(y), e^{x+y}); h(x,y) = g(f(x,y)).$$

Use the Chain Rule to compute the derivative matrix of h at the origin.

4. Is the difference of two stable mappings also a stable mapping?

5. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be continuously differentiable and stable. Prove that for any $x \in \mathbb{R}^3$ and for any $h \in \mathbb{R}^3$, $\|DF(x)h\| \geq \|h\|$.