

MATH 141 Midterm 3

3.) 
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\log_{10} n}}$$

Using the integral test we note that the sequence can be extended to a monotone decreasing non-negative continuous function,  $\frac{1}{x\sqrt{\log_{10} x}}$ , on the interval  $[2, \infty)$ . Thus the series converges

if and only if the integral  $\int_2^{\infty} \frac{dx}{x\sqrt{\log_{10} x}}$  converges. (5pts)

Using the u substitution  $u = \log_{10} x$  and  $du = \frac{dx}{x \ln 10}$  we now take the integral

$$\ln 10 \cdot \int_{\log_{10} 2}^{\infty} \frac{du}{\sqrt{u}} = \ln 10 \cdot [2\sqrt{u}]_{\log_{10} 2}^{\infty} \quad (10\text{pts})$$

Since the  $\lim_{u \rightarrow \infty} \sqrt{u} = \infty$  the integral diverges and thus the series must also diverge. Thus by the integral test the series diverges. (5pts)