MATH 648M: Advanced Analytic Methods with Applications
Department of Mathematics, UMCP **Spring 2008**
Homework 3 **Posted**: Monday, 03/10/08
Due: Thursday, 03/27/08

7. (20 pts) [Exercise on Boundary Layers] Solve approximately the following equations for $0 < \epsilon \ll 1$ by following arguments similar to those given in class. (Solutions that *only* involve applications of theorems *won't be acceptable*):

   (a) $\epsilon u''(x) + (1+x)^2 u'(x) + u(x) = 0,\ u(0) = 0,\ u(1) = 2;\ 0 < x < 1.$

   (b) $\epsilon u''(x) - (1+x^2) u'(x) + u(x) = 0,\ u(0) = 0,\ u(1) = 2;\ 0 < x < 1.$

8. (20 pts) [Boundary Layer for a Volterra integral equation] Consider the integral equation

$$\epsilon u(x) = -q(x) \int_0^x [u(s) - f(s)]s\, ds, \quad 0 \le x \le 1,\ 0 < \epsilon \ll 1,$$

   where $u(x)$ is unknown while f and q are given, sufficiently differentiable functions.

   (a) (10 pts) Take $\underline{q(x) = 1}$: Find a composite expansion for $u(x)$.

   (b) (10 pts) Let $q(x)$ be positive but arbitrary. Find a composite expansion for $u(x)$.

   Hint on (a), (b): You may think in two different ways in order to solve this problem. One way is to convert the given integral equation to a differential equation assuming that u is sufficiently smooth.

9. (20 pts) The Reynolds equation in gas lubrication theory is

$$\epsilon \frac{d}{dx}(H^3 u u') = \frac{d}{dx}(Hu), \quad 0 < x < 1,\ 0 < \epsilon \ll 1,$$

   where $u(x)$ is unknown, $\underline{u(0) = u(1) = 1}$, and $H(x)$ is a known, smooth, positive function with $H(0) \neq H(1)$. Find a composite expansion for $u(x)$. **Hint:** Part or all of the solution will be defined implicitly; but it is still possible to match the expansions.