MATH 648M: Advanced Analytic Methods with Applications
Department of Mathematics, UMCP **Spring 2013**
Homework 5 **Posted**: Thursday, 04/18/13
Due: Tuesday, 05/07/13

If you take the course for 3 credits, do all problems. If you take the course for 1 credit, do Problems 17 and 18. Computers are not allowed.

16. [10pts] Show that the evolution of the quantum state of a particle by the Schrödinger equation leads to the path integral formulation in one spatial dimension. Assume that the potential V depends on time, $V = V(x,t)$, $x \in \mathbb{R}$ (real). **Hint:** Suppose that the particle is at the initial state $|q_i\rangle$ at time T_i. You need to determine the amplitude of finding this particle at the final state $|q_f\rangle$ at time T_f. Split the time interval $[T_i, T_f]$ into N subintervals, each of width ϵ. Consider the projection of the evolved state onto $|q_f\rangle$; and apply suitable superposition over intermediate position eigenstates (assuming these form a basis set). You may find it useful to employ eigenstates of the momentum operator at some stage of your derivation.

17. (a)[10pts] Determine Green's function with Feynman boundary conditions for the Klein-Gordon equation in the presence of an electromagnetic field,

$$-\left(\frac{\partial}{\partial t} + ieA^0\right)^2 \Psi(x) = \left[-\left(\vec{\nabla} - ie\vec{A}\right)^2 + m^2\right] \Psi(x).$$

Write down an integral equation formulation.

(b)[6pts] Is $\Psi^\dagger(x)\Psi(x)$ a conserved quantity for the Dirac equation coupled with an electromagnetic field? Explain. **Note on the notation in 17(a), (b):** $x = (x^0, \vec{x})$.

(c)[4pts] Compute $\not{p}\not{k} + \not{k}\not{p}$ in terms of a scalar product. Find $\mathrm{Tr}(\not{k}\not{p})$.

18. Consider $N+1$ particles, each having mass m and being joined with springs of constant κ to its nearest neighbors. The particle coordinates in the x-axis are fixed to $n\epsilon$ where $n = 0, 1, \ldots, N$. Suppose the particles are allowed to move in *both* the y-direction and the z-direction. Let the displacements of the nth particle in these directions be denoted by φ_n and ψ_n, respectively.

(a)[5pts] Express the classical Lagrangian of the system in terms of $\phi_n := (\varphi_n + i\psi_n)/\sqrt{2}$ and its complex conjugate. Derive the Euler-Lagrange equations. Determine the Hamiltonian of this system.

(b)[5pts] Let $\epsilon \downarrow 0$. Obtain the Lagrangian and the Hamiltonian density of this system. What are the governing equations of motion in this limit?

(c)[10pts] Let the string of particles be of infinite length. Quantize this field theory, and find the creation and annihilation operators of this system. Express the Hamiltonian and momentum of the system in terms of these operators. Find the eigenvalues and corresponding eigenstates.