MATH 648M: Advanced Analytic Methods with Applications
Department of Mathematics, UMCP **Spring 2013**
Handout 1: COURSE SYLLABUS AND POLICIES

Lecture Room: MATH 0201 **Time:** TuTh 9:30a.m.– 10:45a.m.

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Office hours: Thursdays 11:00a.m.-12:00p.m.; or by appointment. Since I am occasionally called away for research matters, it is advisable to check with me as to whether I will be in my office at the indicated hours.

Prerequisites: None, typically. The course is largely self-contained but also requires some self-study on the way. Some knowledge of complex variables and differential equations will be very helpful. The UMD courses MATH 414, 462, or 463, for example, would suffice, but they are not required. Handouts with reviews will be distributed in class. Ask for the instructor's permission if you are in doubt.

Text: No required text. A few recommended texts are in the suggested Bibliography (Handout 2). The actual syllabus (below) corresponds to less than half the material in these texts.

Course Web page: http://www.math.umd.edu/∼dio/courses/648M/
All homeworks and practice problems and some solutions will be posted there.

Teaching Assistant/Grader: TBA, if any.
Grading policy: NO exams or tests. Grades will be based exclusively on 5-6 problem sets. Each set will be due (usually) 2 weeks after the date it is handed out.

You can take the course for 1-3 credits. You are expected to write your *own* solutions.

Once assigned, the homeworks must be turned in by the date specified. Your solutions are required to be *legible and clear*. Illegible returned homeworks will not be graded.

Since the course aims to sharpen your ingenuity and analytical skills, there is no need for calculators for the homeworks. Accordingly, calculators are *prohibited*.

Scope and topics: The course encompasses concepts and analytic techniques that: (i) permeate classical mechanics as well as quantum mechanics and quantum field theories; and (ii) aim to help students formulate and solve problems in the physical sciences and applied mathematics, from the sub-atomic to the macroscopic scale. Applications span fluid and solid mechanics, elasticity, electromagnetism, and condensed-matter, atomic and particle physics. There are two main parts:

PART I. This part includes methods that originated from classical mechanics (and can, heuristically or rigorously, be invoked in quantum theories). A tentative list of topics follows.

Theory: Motivation. Some background. Operators in Hilbert space: Euclidean (vector) space; Hilbert space; formal definitions; and results; the Fredholm alternative.

Green's function: Definition; solution of boundary value problems in mechanics. Integral equations: Introduction and simple examples; Fredholm equations of second and first kind;

the Wiener-Hopf method.

Lagrangian formulation: Action principle; Lagrangian and Hamiltonian approaches; Euler-Lagrange and Hamilton's equations of motion; examples; variational principles of mechanics. Symmetries and conservation laws: An example; Noether's theorem; dispersive systems.

Perturbation theory: An example from diffraction of waves; the Born-Neumann series and convergence issues; extensions and generalizations; bifurcations.

Elements of special relativity.

Applications (partly in homeworks): How does light scatter from a metallic strip? How can one estimate the air flow past an airplane wing? What is the shape of a laser pulse in an optical fiber? What is the elastic strain caused by defects on a crystal surface? And more...

PART II. This part primarily addresses quantum theories via methods learned above.

Theory: Elements of quantum mechanics: Wave function; Schrödinger's equation; Dirac's equation; the Klein-Gordon equation. Second quantization; Bosons and Fermions.

Quantum electrodynamics (QED): Formalism; spin and statistics; radiation field and its quantization; free electrons, spinors and their quantization; interaction of radiation with electrons; gauge transformations. Beyond QED: the SU(2) Yang-Mills gauge field theory; extensions; the Higgs mechanism; the Standard Model.

Calculus of Feynman diagrams: Dyson's series; the S-matrix; divergencies; analytical properties; dimensional regularization; high-energy asymptotics.

Applications: What is the amount of impurities in a transistor? How do atoms of integer spin condense in a trap at very low temperatures? How can one derive the Coulomb interaction of two electrons? How do total scattering cross sections behave at very high energies? □

Note on Academic Integrity. You are expected to read carefully and adhere to the following instruction provided by the Student Honor Council.

The University of Maryland, College Park has a nationally recognized Code of Academic Integrity, administered by the Student Honor Council. This Code sets standards for academic integrity at Maryland for all undergraduate and graduate students. As a student you are responsible for upholding these standards for this course. It is very important for you to be aware of the consequences of cheating, fabrication, facilitation, and plagiarism. For more information on the Code of Academic Integrity or the Student Honor Council, please visit http://www.shc.umd.edu.

To further exhibit your commitment to academic integrity, remember to sign the Honor Pledge on all assignments: *"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment."*

Additional note for MATH 648M: You will not be asked to sign such a pledge on possible homework assignments, but you are nevertheless expected to adhere to the principles of the pledge. The rationale for the pledge is available online at http://www.umd.edu/honorpledge.

Students With Disabilities. If you have a documented disability and need academic accommodations, please contact me as soon as possible.

Religious Observances. If you will be absent from class because of religious observances, please submit a list of the dates of your absences within a couple of days.