

Exam 1

Handed out to class: Monday, 09/25/17

READ CAREFULLY AND WORK ON ALL PROBLEMS. Justify your answers. Show all your steps.
Cross out what is not part of your final answer. NO calculators or textbooks or notes are allowed.
Total regular time: 50min.

1. [10 pts] Consider the vectors $\mathbf{a} = \mathbf{i} - \lambda\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mu\mathbf{i} + \lambda\mathbf{j} + 5\mathbf{k}$, where λ and μ are real numbers.
- (a)[5pts] Compute the dot product and the cross product of \mathbf{a} and \mathbf{b} for any real λ and μ .
- (b)[5pts] By using part 1(a), find the values of λ and μ such that \mathbf{a} is parallel to \mathbf{b} .

2. [10 pts] Consider the lines ℓ_1 and ℓ_2 described by the following equations:

$$\ell_1: \frac{x-1}{3} = \frac{y-2}{2} = z-3, \quad \text{and} \quad \ell_2: \frac{x-3}{2} = y-3 = \frac{z-6}{3}.$$

- (a)[6pts] Show that lines ℓ_1 and ℓ_2 intersect and find the point of their intersection.
- (b)[4pts] Find the equation of the plane \mathcal{P} that contains both ℓ_1 and ℓ_2 . **Note:** You may use the result of 2(a).

3. [10 pts] The space curve C is parametrized by the vector-valued function

$$\mathbf{r}(t) = \frac{1}{3}(\sin t)^3 \mathbf{i} + 2\mathbf{j} + \frac{1}{3}(\cos t)^3 \mathbf{k}, \quad a \leq t \leq b \quad (\text{where } a < b).$$

- (a)[4pts] Let $a = -\pi/4$ and $b = \pi/4$. Is C a smooth or a piecewise smooth curve? Explain.
- (b)[6pts] Let $a = 0$ and $b = \pi/4$. Determine the length L of the curve C . **Hint:** You may use $\sin t \cos t = (1/2) \sin(2t)$.

4. [10 pts] A fly named Harry moves along the trajectory C_1 parametrized by

$$\mathbf{r}_1(t) = \frac{1}{4}e^{4t} \mathbf{i} - \frac{1}{\sqrt{2}}e^{2t} \mathbf{j} + t \mathbf{k} \quad \text{for } t \geq 0.$$

- (a)[2pts] Find the velocity and acceleration of Harry as a function of time, t .
- (b)[6pts] Compute the tangential and normal components, a_T and a_N , of Harry's acceleration.
- (c)[2pts] Another fly, Sally, moves at the same time t along the trajectory C_2 parametrized by

$$\mathbf{r}_2(t) = \frac{1}{4}e^{t^3} \mathbf{i} + \frac{1}{\sqrt{2}}e^{t^2} \mathbf{j} + \frac{1}{2}t^2 \mathbf{k} \quad \text{for } t \geq 0.$$

Will Harry meet Sally at any time $t > 0$, or not? Explain.