MATH 241 (Sec. 03xx): CALCULUS III
Department of Mathematics, UMCP **Fall 2017**

Exam 2

Handed out to class: Monday, 10/23/17

READ CAREFULLY AND WORK ON ALL PROBLEMS. Justify your answers. Show all your steps. Cross out what is not part of your final answer. NO calculators or textbooks or pre-prepared notes are allowed. Total regular time: 50min.

1. (10 pts) Consider the function $z = y^{-1}e^{-x/y}$ for $y \neq 0$. Show that z satisfies the following equation:

$$\frac{\partial z}{\partial y} = x\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x} \ .$$

2. (a)(5 pts) Consider the function $f(x, y) = \sin(xy)$. Find the direction in which f increases most rapidly at the point $(x_0, y_0) = (1, \pi)$. What is the maximal directional derivative of f at this point?

   (b)(5 pts) Now consider the function $f(x, y, z) = \sin(xy) + \cos(xz)$. Find an equation for the plane tangent to the level surface $f(x, y, z) = 0$ at the point $(x_0, y_0, z_0) = (\pi/4, 4, 2)$.

3. (10 pts) Consider the function $f(x, y) = -2x^2 + 3xy + y^2 - 4x + 3y - 1$.

   (a)(6 pts) Find all critical points of f. Specify which of these points correspond to relative maximum values, relative minimum values and saddle points of f. **Hint:** Apply the second partials test.

   (b)(4 pts) Suppose that f takes values on the region R (circular disk) defined by $x^2 + y^2 \leq 4$. Explain why f has a maximum and a minimum value in R. Are such values attained in the interior or at the boundary of R? Explain. **Note:** You are *not* asked to find these values.

4. (10 pts) Consider the function $w = f(y - x + a, y - z + b, z - x + c)$ where f is an arbitrary function of three variables, and a, b and c are constants. Assume that the partial derivatives of f exist. Compute the following sum:

$$A = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} \ .$$