MATH 241 (Sec. 03xx): CALCULUS III
Department of Mathematics, UMCP **Fall 2017**

Exam 3

Handed out: Wednesday, 11/15/17

READ CAREFULLY AND WORK ON ALL PROBLEMS. Justify your answers. Show all your steps. Cross out what is not part of your final answer. NO calculators or textbooks or pre-prepared notes are allowed. Total regular time: 50min.

1. (10 pts) Let R be the trapezoidal region bounded by the lines $y = 1$, $y = 2$, $y = (1/2)x$ and the positive y axis. Compute the double integral
$$\mathcal{M} = \iint_R y\, dA\ .$$

2. (10 pts) By using a double integral and polar coordinates, find the volume V of the solid region bounded above by the plane $z = 4 + x + 2y$, on the sides by the cylinder $x^2 + y^2 = 1$, and below by the xy plane. **Note:** Define V as a double integral to get credit.

3. (10 pts) An object occupies the solid region D between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$, and above the upper nappe of the cone $3z^2 = x^2 + y^2$. The mass density $\delta(x, y, z)$ of this object at any point (x, y, z) of D is equal to the distance of this point from the origin.

   (a)[2 pts] Write down a triple integral that gives the total mass m of the object in region D.

   (b)[8 pts] Compute the mass m from part 3(a). **Note:** For full credit, simplify your answer as much as possible. Recall that $\tan(\pi/3) = \sqrt{3}$.

4. (10 pts) Consider the integral
$$I = \iint_R \left(1 + \frac{x^2}{16} + \frac{y^2}{9}\right)^{5/2} dA\ ,$$
where R is the region of the xy-plane bounded by the ellipse $x^2/16 + y^2/9 = 1$.

   (a)[3 pts] Find a transformation $x = g_1(u, v)$, $y = g_2(u, v)$ that transforms the ellipse $x^2/16 + y^2/9 = 1$ to the unit circle (i.e., the circle of radius equal to 1) in the uv-plane.

   (b)[7 pts] Compute I by changing variables of integration according to part 4(a). **Note:** For full credit, simplify your answer as much as possible.