MATH 241: CALCULUS III
Department of Mathematics, UMCP **Fall 2010**

Practice Exam 1

Posted: Saturday, 09/18/10

WORK ON ALL PROBLEMS. Justify your answers. Cross out what is not meant to be part of your final answer.

1. (a)[5 pts] Suppose **a** and **b** are nonzero vectors, and θ is the angle between these vectors. Show that $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta$. **Hint:** Apply the law of cosines to the triangle formed by **a**, **b** and $\mathbf{a} - \mathbf{b}$.

   (b)[2 pts] Consider the vectors

   $$\mathbf{a} = \mathbf{i} + \lambda\,\mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \mathbf{j},$$

   where λ is a real number. Find λ so that **a** and **b** are perpendicular.

   (c)[3 pts] For the vectors **a**, **b** of part 1(b) and the value of λ found in part 1(b), resolve $\mathbf{c} = \mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$, into vectors parallel to **a** and **b**.

2. Consider the following vectors:

   $$\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \mathbf{k}\ .$$

   (a)[6 pts] Find the equation of the plane that contains **a** and **b** and goes through the point $P = (1, 0, -1)$.

   (b)[4 pts] Find the point at which the plane found in part 2(a) intersects the line $x = 3t$, $y = 1 + 2t$, $z = -1 + t$.

3. (10 pts) Let C be the curve traced out by the vector function

   $$\mathbf{r}(t) = \ln t\,\mathbf{i} + \frac{t^2}{2}\mathbf{j} + t\sqrt{2}\,\mathbf{k}\ , \quad 1 \le t \le 2\ .$$

   Find the arc length function $s(t)$ for $1 \le t \le 2$. What is the total distance traveled by a particle on C for $1 \le t \le 2$?

4. (10 pts) A curve C is traced out by the vector function

   $$\mathbf{r}(t) = (t + \sin t)\mathbf{i} + (1 + \cos t)\mathbf{j} + 4\cos(\tfrac{t}{2})\,\mathbf{k}\ .$$

   Find the unit tangent and unit normal vectors **T** and **N** of the curve C (as functions of t).