MATH 463: Complex Variables for Scientists and Engineers
Department of Mathematics, UMCP **Spring 2015**
Exam 1 **Handed out**: Friday, 02/27/15

Answer all questions. Make sure that you explain all your steps and justify your answers.

1. (a)[4pts] Show that, for $n \geq 1$,

$$1 + z + z^2 + \ldots + z^n = \frac{z^{n+1} - 1}{z - 1}\,, \qquad z \neq 1.$$

(b)[6pts] Compute the sum

$$1 + \cos\theta + \cos(2\theta) + \ldots + \cos(n\theta)$$

in terms of n and θ, where $n \geq 1$ and $0 < \theta < 2\pi$.

2. (a)[1pt] Give the definition of a harmonic function.

(b)[2pts] Show that if $f(z) = u + iv$ is analytic in the domain $\mathcal{D}$, then each of u and v is harmonic in $\mathcal{D}$. Assume that u and v have continuous second partial derivatives in $\mathcal{D}$. **Hint:** You may use the fact that u and v satisfy the Cauchy-Riemann equations.

(c)[3pts] Show that the function $v(x, y) = 3x^2y - y^3 + x^2 - y^2$ is harmonic for all (x, y).

(d)[4pts] Determine the function $u(x, y)$ such that $f = u + iv$ is entire, where v is given in 2(c) above.

3. [10pts] Find all complex z that satisfy the equation $\sin z = -i\lambda \cos z$ where λ is real and $0 < \lambda < 1$. **Hint:** Express this equation in terms of $e^{iz} = w$ and first solve for w^2.

4. [10pts] Find the partial fraction decomposition of the following rational functions:

(a)[5pts] $f(z) = \dfrac{2 + i}{z(z+1)(z+3)}$

(b)[5pts] $f(z) = \dfrac{2i}{(z^2+1)^2}$