MATH 463: Complex Variables for Scientists and Engineers
Department of Mathematics, UMCP   **Spring 2015**
Exam 2   **Handed out**: Monday, 04/13/15

Answer <u>all</u> questions. Make sure that you <u>explain</u> all your steps and <u>justify</u> your answers.

1. Consider the function
$$f(z) = \frac{z^2 + z - 1}{z(z-1)^2} \ .$$

   (a)[2pts] Find and classify <u>all</u> singular points of $f(z)$.

   (b)[8pts] Compute the integral
$$I = \int_\Gamma f(z)\,\mathrm{d}z \ ,$$

   where the closed contour Γ is traversed once positively, in the following cases:
   b.i) Γ is the circle with center at $z = 2$ and radius $3/2$;
   b.ii) Γ is circle with center at $z = 2$ and radius 100.

2. Consider the integral
$$H(z) = \frac{1}{2\pi i} \int_C \frac{e^\zeta + \zeta^{-1}}{\zeta - z}\,\mathrm{d}\zeta \ ,$$

   where C is the circle with center at $z = 0$ and radius 1, traversed once counterclockwise. Compute the following values:

   (a)[4pts] $H(0)$;   (b)[6pts] $\lim_{z \to i} H(z)$ if z lies <u>outside</u> C.

3. Consider the function
$$f(z) = \frac{z-1}{3-z} \ .$$

   (a)[8pts] Find the Taylor series for $f(z)$ at $z_0 = 0$. What is the radius of convergence of this Taylor series? Explain carefully.

   (b)[2pts] Consider the function $g(z) = e^{f(z)}$. What kind of isolated singularity of $g(z)$ is the point $z_0 = 3$? Explain carefully.

4. [10pts] Consider the function
$$f(z) = \frac{1}{z(z-i)} \ .$$

   Find the Laurent series for $f(z)$ at $z_0 = 0$ in the annulus $1 < |z| < \infty$.