NAME: SOLUTIONS

MATH463, Sec. 0101: In–class Quiz # 1
Wednesday, February 18, 2015

Solve the following 2 problems. Justify your answers. Cross out what is not meant to be part of your final answer. Total number of points: 10.

I. (5 pts) Find *all* complex values of

$$(1-i\sqrt{3})^{2/5}.$$

$$1-i\sqrt{3} = \sqrt{1+(\sqrt{3})^2}\; e^{i\theta_0+i2k\pi} = 2\; e^{i\theta_0+i2k\pi}$$

where $\theta_0 = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\pi/3$

Thus, $1-i\sqrt{3} = 2\; e^{-i\pi/3+i2k\pi}$, $k=0,\pm1,\pm2,\dots$

$$(1-i\sqrt{3})^{2/5} = \underbrace{\sqrt[5]{4}}_{>0}\; e^{\frac{2}{5}\left(-i\frac{\pi}{3}+i2k\pi\right)}$$

$$= \sqrt[5]{4}\; e^{-i\frac{2\pi}{15}+i\frac{4k\pi}{5}}, \quad k=0,1,2,3,4.$$

CONTINUED ON REVERSE

II.(5pts) Consider the function $f(z) = u(x,y) + iv(x,y)$ where

$$u(x,y) = x^2 - y^2 + 2y + x \ , \qquad v(x,y) = 2xy - 2x - y$$

(a)[3 pts] Is $f(z)$ an entire (i.e., analytic everywhere) function? Explain.

(b)[2 pts] Write down explicitly $f(z)$ in terms of z and possibly $\bar{z}$. **Note:** If $f(z)$ <u>is</u> analytic, it should involve only z and <u>not</u> $\bar{z}$. If $f(z)$ <u>is not</u> analytic, it should involve <u>both z and $\bar{z}$</u>.

(a) Check Cauchy-Riemann eqns: One of them is:

$$\frac{\partial u}{\partial x} = 2x+1 \neq \frac{\partial v}{\partial y} = 2x-1$$

Not satisfied. Thus, $f(z)$ is <u>not</u> entire.

[You may also check: $\frac{\partial u}{\partial y} = -2y+2 = -\frac{\partial v}{\partial x} = -2y+2$: satisfied!]

(b) Let $x = \frac{z+\bar{z}}{2}$, $y = \frac{z-\bar{z}}{2i}$.

$$f(z) = x^2-y^2+2y+x+i(2xy-2x-y)$$

$$= \underbrace{x^2-y^2+i2xy}+2y+x-i2x-iy$$

$$= (x+iy)^2 - i2(x+iy) + x - iy$$

$$= z^2 - 2iz + \bar{z} \qquad \text{(Not analytic)}$$